FLEXURAL DESIGN OF STRAIN HARDENING CEMENT COMPOSITES

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Abstract

A parametric model for simulation of tensile behaviour of cement-based composites is used to correlate the stress strain constitutive response with flexural load carrying capacity of strain hardening cement composites. This procedure can also be used as a design approach. Using a back-calculation approach, the results of tensile experiments of composites are converted to a parametric model of strain hardening material and closed form equations for representation of flexural response of sections are obtained. Results are then implemented as average moment-curvature relationships in the structural design and analysis of one way and two way flexural elements using yield line analysis approaches. Correlation of material properties with simplified design equations is shown. Independent experimental results obtained in-house and from literature are used to verify the model using textile Reinforced Concrete with alkali resistant (AR) glass textile reinforced concrete and ECC Composites. A generalized approach for the design equations using a plastic analysis approach is shown

1. INTRODUCTION

A significantly high level of strength, ductility, and versatility has been attained in the field of Strain Hardening Cement Composites (SHCC) recently as demonstrated by novel materials including Textile Reinforced Concrete (TRC) and Engineered Cementitious Composites (ECC) [1]. With as much as one order of magnitude higher strength and two orders of magnitude higher in ductility than fiber reinforced concrete (FRC), TRC’s development has utilized innovative fabrics, matrices, and manufacturing processes. A variety of fiber and fabric systems such as Alkali resistant (AR) glass fibers, polypropylene (PP), polyethylene (PE), and Poly Vinyl Alcohol (PVA) have been utilized [2-4]. Mechanical properties of the composites under uniaxial tensile, flexural, and shear tests indicate superior performance such as tensile strength as high as 25 MPa, and strain capacity of 1-8%. In order to fully utilize these materials, design guidelines are needed to determine the dimensions and expected load carrying capacity of structural systems. This paper presents an approach for the design of strain hardening cement composite systems applicable to both TRC and ECC.
2. STRAIN-HARDENING FIBER REINFORCED COMPOSITE MODEL

The behavior of SHCC systems can be predicted with a tri-linear model representing tensile strain hardening tensile response and an elastic perfectly plastic compression model [5]. As shown in Figure 1, by normalizing all parameters with respect to minimum number of variables, tensile response is defined by stiffness, $E$, first crack tensile strain $\varepsilon_{cr}$, cracking tensile strength, $\sigma_{cr} = E \varepsilon_{cr}$, ultimate tensile capacity, $\varepsilon_{peak}$, and post crack modulus $E_{cr}$. The softening range is shown as a constant stress level $\mu E \varepsilon_{cr}$. The compression response is defined by the compressive strength $\sigma_{cy}$ defined as $\omega \gamma E \varepsilon_{cr}$. Material parameters required for the Strain Softening (SSCC) and SHCC are summarized as follows. Parameters, $\alpha$, $\mu$, $\eta$, $\omega$ are defined respectively as normalized tensile strain at peak strength, post-crack modulus, and compressive yield strain and applied tensile and compressive strains at bottom and top fibers, $\beta$ and $\lambda$ are defined as:

$$\alpha = \frac{\varepsilon_{peak}}{\varepsilon_{cr}}, \quad \eta = \frac{E_{cr}}{E}, \quad \omega = \frac{\sigma_{cy}}{\sigma_{cr}}, \quad \beta = \frac{\varepsilon_{t}}{\varepsilon_{cr}}, \quad \lambda = \frac{\varepsilon_{c}}{\varepsilon_{cr}}$$

The ratio of compressive and tensile modulus, parameter $\gamma$, has negligible effect on the ultimate moment capacity [6]. In typical SHCC, the compressive strength is several times higher than tensile strength, hence the flexural capacity is controlled by the tensile component.

Moment capacity of a beam section according to the imposed tensile strain at the bottom fiber ($\varepsilon_{t} = \beta \varepsilon_{cr}$) can be derived based on the assumption of linear strain distribution. By using material models described in Figure 1(a) and (b), force components, their centroidal distance to the neutral axis, the moment, and curvature distributions are obtained. The depth of neutral axis, the nominal moment capacity $M_n$ are also obtained and expressed as a product of the normalized nominal moment $m_n$ and the cracking moment $M_{cr}$. The neutral axis parameter $k$ is found by solving the equilibrium of net internal forces and the nominal moment capacity $M_n$ is obtained by taking the first moment of force about the neutral axis as shown in Eqs. 2 and 3.

$$k = \frac{C_1 - \sqrt{\beta^2 C_1}}{C_1 - \beta^2}; \quad \text{where} \quad C_1 = \eta (\beta^2 - 2 \beta + 1) + 2 \beta - 1$$

$$M_n = m_n M_{cr}; \quad m_n = C_2 \frac{k^2 - 2k + 1}{\beta^2} \sqrt{\frac{2 \beta k^3}{1 - k}}; \quad C_2 = C_1 + 2C_1 \beta - \beta^2, M_{cr} = \frac{\sigma_{cy} bh^2}{6}$$

Location of Neutral axis and moment capacity are presented in Table 1 with all potential combinations for the interaction of tensile and compressive failure responses. The maximum moment capacity is obtained when the normalized tensile strain at the bottom fiber ($\beta = \varepsilon_{t}/\varepsilon_{cr}$) reaches the tensile strain at peak strength ($\alpha = \varepsilon_{peak}/\varepsilon_{cr}$).

The general moment-curvature profile as shown in the parametric studies indicates that fiber contribution is mostly apparent in the post cracking tensile region, (Figure 1(a)). Parametric analysis of these equations indicates that the response governed by the post-crack modulus $E_{cr}$ is relatively flat with values of $\eta = 0.00-0.4$ for a majority of cement composites. Tensile strain at peak strength $\varepsilon_{peak}$ is relatively large compared to the cracking tensile strain $\varepsilon_{cr}$ and may be as high as $\alpha = 100$ for systems with polymeric fibers. These characteristics cause the flexural strength to continue to increase after cracking. Since typical SHCC may not
have a significant post-peak tensile strength, the flexural load drops after the tensile strain at
peak strength, hence post crack tensile parameter $\mu$ may be ignored for simplified analysis [2].

![Material model for SHCC and SSCC FRC: (a) compression; and (b) tension model.](image)

Figure 1: Material model for SHCC and SSCC FRC: (a) compression; and (b) tension model.

Steps in calculation of load-deflection response from the moment-curvature have been
discussed in detail in recent publications dealing with SHCC [7] and softening composites [8].
The load–deflection response of a beam can be obtained by using the moment–curvature,
crack localization rules, and moment-area method.

3. PARAMETRIC STUDIES OF LOAD DEFLECTION RESPONSE

Parametric studies evaluate the effect of different parameters on the moment-curvature and
load deflection response. Due to the nature of modeling, a unique set of properties from the
flexural tests can’t be obtained as there are a range of tensile properties that may result in
similar flexural responses. It is therefore essential to measure and match both tension and
flexural responses in the back-calculation processes. Figures 2, 3, and 4 show the effect of
model parameters $\mu$, $\alpha$, and $\eta$ on the simulated response. These key parameters are changed to
best fit the experimental load-deflection and tensile stress strain curves. Note that simulations
that use direct tension data underestimate the equivalent flexural stress. This may be due to
several factors including size effect, uniformity in tension loading vs. the linear strain
distribution in flexure, and variation in lamina orientation which may lead to a wider variation
in flexural samples. The underestimation of flexural capacity can be reduced by applying
scaling parameters to the tensile capacity [7]. This procedure has potential for use of the
flexural data to develop the moment-curvature response for the design of flexural load cases.
Figure 2: Parametric study of parameter $\mu$ as shown in (a) load deflection and (b) stress strain.

Figure 3: Parametric study of parameter $\alpha$ in (a) load deflection and (b) stress strain.

4. LOAD DEFLECTION RESPONSE OF FABRIC CEMENT COMPOSITES

Two types of composites consisting of TRC composites and ECC materials were used. Three different TRC composites consisting of upper and lower bound AR-Glass with alternate 100 lb or 200 lb of confinement pressure and/or the addition of 40% fly ash were used [3,9]. Composites were manufactured using a cement paste with a w/c=0.45, and 8 layers of AR-Glass manufactured by Nippon Electric Glass Co. [3]. Both experimental data from a set of specimens under uniaxial tension and three point bending tests were used, however no attempt was made to obtain a best fit curve. Tensile TRC composites were 10x25x200 mm. Flexural three point bending samples were 10x25x200 mm with a clear span of 152 mm. Material parameters were determined by fitting the hardening model to both tension and flexural tests. Results are shown by the simulated upper and lower bounds encompassing the TRC’s in
Figure 4: Parametric study of parameter $\eta$ in (a) load deflection and (b) stress strain.

Figures 5 (a) and (b). Figure 5 (a) shows the predicted flexural load deflection response and Figure 5 (b) shows the tensile stress strain responses compared with experimentally obtained results. Representative properties for the simulation of upper bound values obtained from the GNS200 samples were: $\alpha=50$, $\mu=3.9$, $\eta=0.06$, $\gamma=5.0$ and $\omega=10$ the constants were $\varepsilon_{cr}=0.0002$, and $E=20000$ MPa, while the limits of the modeling were $\beta_{tu}=135$, and $\lambda_{cu}=40$. Representative material properties for the lower bound values from the GFA40 samples were: $\alpha=32$, $\mu=2.0$, $\eta=0.032$, $\gamma=5.0$ and $\omega=10$ the constants were $\varepsilon_{cr}=180$ $\mu$str, and $E=20$ GPa, while the limits of the modeling were $\beta_{tu}=150$, and $\lambda_{cu}=40$. Values shown apply to typical set of data and proper optimization with upper and lower bound values are required.

An ECC mix utilizing 2.0% by volume of polyethylene (PE) fibers from the literature [10,11] was also modeled. The flexural specimens for the four point bending test were 76x101x355 mm with a clear span of 305 mm [12]. Load–deflection response is shown in Figures 6 (a) and (b).

Table 2: Data from experimental analysis of representative TRC and ECC samples.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>b (mm)</th>
<th>d (mm)</th>
<th>L (mm)</th>
<th>Flexural Stiffness (N/mm)</th>
<th>Defl @ Max Flex Load (mm)</th>
<th>Max Flex Load (N)</th>
<th>Bending Stress (MPa)</th>
<th>Flexural Toughness N.mm/mm²</th>
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<tr>
<td>GNS200</td>
<td>30</td>
<td>9</td>
<td>152</td>
<td>801</td>
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<td>520</td>
<td>49</td>
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<tr>
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<td>7</td>
<td>152</td>
<td>565</td>
<td>6.25</td>
<td>138</td>
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<tr>
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<td>8</td>
<td>152</td>
<td>510</td>
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<td>26</td>
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<tr>
<td>ECC-PE2</td>
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<td>102</td>
<td>305</td>
<td>60351</td>
<td>6.23</td>
<td>30114</td>
<td>12</td>
<td>26.36</td>
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Table 3: Material model parameters from back calculation of TRC and ECC samples.

<table>
<thead>
<tr>
<th>Sample ID</th>
<th>Young’s Modulus (E) GPa</th>
<th>Young’s Modulus (E) GPa</th>
<th>Crack Strength ($\sigma_{cr}$) MPa</th>
<th>Crack Strength ($\sigma_{cr}$) MPa</th>
<th>Post Crack Tensile Strength ($\mu$) MPa</th>
<th>Post Crack Tensile Strength ($\mu$) MPa</th>
<th>Transition Tensile Strain ($\alpha$) %</th>
<th>Transitional Tensile Strain ($\epsilon_{trn}$), %</th>
<th>Ultimate Strain ($\epsilon_{tu}$), %</th>
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<tr>
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<td>4.62</td>
<td>2.3</td>
<td>70</td>
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<tr>
<td>ECC-PE2</td>
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<td>5.7</td>
<td>7.8</td>
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</table>

Figure 5: Strain hardening model of TRC (a) Equivalent Flexural Stress Deflection and corresponding (b) Stress Strain response.

Figure 6: (a) Stress-Strain response and (b) Equivalent Flexural Stress Deflection of ECC.
5. DESIGN GUIDELINES – SQUARE, SIMPLY SUPPORTED SLAB

The methodology used in the design of concrete using a plastic analysis approach is used [13]. The nominal moment capacity of a flexural member $M_R$ must be decreased by a reduction factor to account for variability in materials and workmanships. The reduced capacity must be greater than the ultimate moment $M_u$ due to factored loading by:

$$\phi_R M_R \geq M_u$$  \hspace{1cm} (4)

where $\phi_R$ is the reduction factor for strain-hardening FRC and may be taken as 0.9, as stipulated by ACI Sec. C.3.5 [13]. Since the post-crack flexural response of TRC and ECC is ductile, it can sustain large deflections after cracking. Flexural capacity of a simply supported slab subjected to distributed loads is developed (i.e. distributed loading as shown in Figure 7(a)), one can use the principal of virtual work to equate the external and internal work measures. Plastic analysis approach uses the principal of virtual work to equate the internal and external dissipated work to obtain the collapse load. For the example of a distributed load on a simply supported square slab as shown in Figure 7, the work equations are derived as:

$$W_{\text{in}} = W_{\text{out}} = \sum (N_x \times \delta) = \sum (M \times L \times \theta)$$  \hspace{1cm} (5)

Where the resultant four segments of $N_R$ and rotation $\theta$ (from figure 9a, $N_R$ acting at 1/3 of $\delta_{\text{max}}$) and solving Equation (5) for the moments gives:

$$N_x = q \times \left( \frac{L}{2} \right) \times \left( \frac{L}{2} \right) = qL^2, \hspace{0.5cm} \theta = \frac{2\delta_{\text{max}}}{L} \hspace{0.5cm} M_e = \frac{qL^2}{24}$$  \hspace{1cm} (6)

Using moment vs. allowable load relationship one can refer back to equation 5, and depending on the applied load which determines the magnitude of $q_{\text{ult}}$, compute a required ultimate moment capacity and use this equation in conjunction with Equation 4 compute the required section size to carry the given load.

Figure 7: Simply supported square slab with (a) yield lines and (b) loading and rotation conditions through section A-A.
6. CONCLUSIONS

The model presented can be used to simulate the strain hardening behavior of TRC and ECC. By reproducing the experimental data, the moment curvature response of strain hardening materials can be used to predict its moment capacity. This has design implications, as yield line theory can be used in conjunction with the model outputs to generate the ultimate moment capacity for a given slab geometry and loading conditions.

REFERENCES


