EVOLUTION OF ENDOGENOUS SHRINKAGE OF PORTLAND CEMENT PASTES

A. Brahma
Saad Dahlab University, Department of Civil Engineering, BP270 Blida, Algeria

Abstract
In general, the cement paste is the only active phase of hydraulic mixtures. The behavior of cementitious materials is closely related to the evolution of cement pastes. The deformations at early ages may, under certain conditions, cause premature cracking of materials which can compromise performance and durability of structures in service. One of the factors through which the behavior of cement pastes can be analyzed is the degree of hydration which characterizes the state of the evolution of hydration reactions. In this study, we focus on the endogenous shrinkage of cement pastes. The evolution of shrinkage occurs in two phases governed by two different mechanisms. The first phase, the most important is of chemical origin. It is mainly due to a phenomenon called the Le Chatelier contraction. The second phase is counted from when the material is rigid enough to resist at the contraction due to the phenomenon of capillary depression. In this study we show that the evolution of endogenous shrinkage of cement pastes is influenced mainly by the increase in combined water. The parameters influencing the endogenous shrinkage of cement pastes were considered.

Keywords: cement pastes; combined water; chemical shrinkage, endogenous shrinkage; modeling.

1. INTRODUCTION
At very young ages, cement pastes exhibit large dimensional changes which can lead to premature cracking of cement matrix. During the first hours of hydration, shrinkage mainly corresponds to a chemical shrinkage also called "Le Chatelier contraction". It is the consequence of a negative volume balance overall chemical reactions of hydration. After setting, the stiffness of the cement matrix increases gradually and the deformation caused by chemicals is hampered by the solid skeleton of material. Subsequently endogenous shrinkage evolves differently.
In this work we are interested in shrinkage at early age called chemical shrinkage. In the following we present, in the first, a predictive model showing the rate of evolution of Le Chatelier contraction. Then, in a second step, we compare the model predictions and the evolution of chemical shrinkage gotten experimentally by other authors.
2. **MODELING**

Many experimental results show that the evolution of the chemical shrinkage called "contraction Le Chatelier" is represented by curves in the shape of S (Figure 1).

In mathematics, this type of curves is represented by the following differential equation:

\[ y' = ry(1 - \frac{y}{k}) \quad (1) \]

Whose functions \( f \) solutions of this equation are positive definite on \([0; + \infty[\) satisfying the following two conditions:

\[ y(0) = y_0 \]

\[ y' = ry(1 - \frac{y}{k}) \]

With \( r > 0 \) and \( K > 0 \)
Where "K" and "r" are positive real.

To solve this equation one makes a change of variable.

Let: \( y = \frac{1}{z} \)

The expression (1), which is valid for \( y > 0 \), leads to the differential equation:

\[ Z' = -r(z - \frac{1}{K}) \quad (2) \]

This equation admits a general solution \( g(t) \) is the sum of the homogeneous solution and particular.

The homogeneous solution defined by:

\( (Z-1/K) = 0 \Rightarrow z = 1 / K \)

The particular solution:

\[ z' + rz = 0 \Rightarrow z'/z = -r \]

By integrating both sides yields we obtain:

\[ Z = e^{-r}, e^{ct} \]

Let \( e^{ct} = \lambda \) we obtain: \( Z = \lambda \cdot e^{-rt} \quad (3) \)

So the solution is:

\[ g(t) = \lambda \cdot e^{-rt} + 1/K \quad (4) \]

The function \( f \) must verify:
\[ f(t) = \frac{1}{g(t)} = \frac{K}{1 + \lambda Ke^{-rt}} \]  \hspace{1cm} (5)

The initial condition \( y(0) = y_0 \) led to the unique solution:

\[ f(0) = \frac{K}{1 + \lambda K} = y_0 \Rightarrow \lambda K = (\frac{K}{y_0} - 1) \quad \text{Where} \quad f(t) = \frac{K}{1 + (\frac{K}{y_0} - 1)e^{-rt}} \]  \hspace{1cm} (6)

It is easy to verify that this function is well defined and positive on \([0; + \infty[\]

Indeed, \( 1 + (\frac{K}{y_0} - 1)e^{-rt} = e^{-rt}(e^{rt} + \frac{K}{y_0} - 1) \)  \hspace{1cm} (7)

However for \( r > 0 \) and \( t \geq 0 \) \( e^{rt} \geq 1 \) so \( e^{rt} + \frac{K}{y_0} - 1 \geq \frac{K}{y_0} > 0 \)  \hspace{1cm} (8)

It is easy to verify that it meets the two conditions.
According to the values of \( y_0 \), the function is either constant (for \( y_0 = K \)) or increasing (for \( y_0 < K \)) or decreasing (for \( y_0 > K \))

Let \( a = (\frac{K}{y_0} - 1) \)  \hspace{1cm} (9)

For \( a > 0 \) the curve \( y = \frac{K}{1 + ae^{-rt}} \) is the image by an affine transformation of the sigmoid.

Hence the evolution of the Le Chatelier contraction can be easily represented a function \( f \) of the form:

\[ f(t) = \frac{K}{1 + ae^{-rt}} \]  \hspace{1cm} (10)

Where: \( k, a, r \) – parameters of model depending on the temperature and E/C 
\( t \) -time

The parameter \( K \) is determined by the study of the asymptote.

Applying the inverse logit:

\[ \text{Logit} \quad (y) = \ln \left( \frac{y}{1 - y} \right) \quad \text{to the expression} \quad \frac{f(t)}{K} = \frac{1}{1 + ae^{-rt}} \quad \text{allows for a refined fit.} \]

\[ \text{Logit} = \left( \frac{1}{1 + ae^{-rt}} \right) = \ln \left( \frac{1}{1 + ae^{-rt}} \right) - \ln \left( \frac{1}{1 + ae^{-rt}} \right) = \ln \left( \frac{1}{ae^{-rt}} \right) = rt - \ln (a) \]
The adjustment affine Logit (y) then determines "r" and ln(a) and to infer "a".

3. VALIDATION

A comparison of model predictions with experimental results obtained by P. Mounanga [1] is reported in Figures 2. There is good correlation between model predictions and experimental results.

![Figure 2a](image1)

![Figure 2b](image2)
Figure 2c

Figure 2d

Figure 2e
Figure 2f

Figure 2g

Figure 2h
4. COMPARISON WITH OTHER MODELS

Figure 3 shows a comparison between the developed model and other existing models. We can observe that the model developed conducts to the same results as models Mounanga [1] and Bouasker [2]. However, the model developed is simpler to use.

![Comparison of developed model predictions and those models existing.](image)

5. CORRELATION BETWEEN CHEMICAL SHRINKAGE AND THE DEGREE OF HYDRATION

Figure 4 shows the evolution of chemical shrinkage of cement pastes according to the evolution of the degree of hydration. We can observe a linear relationship between this two phenomenon.

We can say that the evolution of endogenous deformation of cement pastes is caused mainly by the movement of water caused by the increase in combined water.

![Correlation between chemical shrinkage and the degree of hydration. W/C=0,25](image)
6. CONCLUSIONS

The results obtained show that the model can predict the progress of the Le Chatelier contraction (the evolution of chemical shrinkage) very satisfactorily. In addition, the conceived model can be adapted to the total prediction of the evolution of the endogenous shrinkage.

Indeed, the deformation of cementitious materials has different origins. In the absence of mass exchange between the material and the external environment, then they are classified as endogenous. They are intimately related to the state of water in the porosity and the progress of hydration reactions. In general, the endogenous shrinkage is caused by an imbalance in water and / or movement of water within the material.

REFERENCES