A FUZZY PROBABILISTIC DURABILITY CONCEPT FOR STRAIN-HARDENING CEMENT-BASED COMPOSITES

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Summary: To fully utilise the advantageous properties of high-performance fibre-reinforced materials, a performance-based durability design concept is required. Probabilistic approaches developed for ordinary crack-free concrete provide a rational basis for this. However, they have to be extended due to a lack of data required to quantify the input parameters and the need to adapt the underlying analytical formulas describing corrosion processes. These formulas have to account for material-specific conditions and the resultant behaviour such as multiple cracking in case of strain-hardening cement-based composites (SHCC).

In this paper a fuzzy probabilistic concept to assess the durability of structural members made of SHCC is presented. Although only exposure to chlorides is explicitly considered, this concept may be regarded as a general framework for durability concepts for novel types of cementitious materials and different forms of environmental action. The analytical solution for chloride ingress used in the DuraCrete approach was adapted to allow for a mathematically correct description of the influence of ageing and to clearly reflect the contribution of cracks to chloride ingress. Furthermore, the considerable non-stochastic uncertainty associated with most parameters in a new material is accounted for with the help of fuzzy-probability theory. Furthermore, the transparent and reproducible quantification of input variables based limited experimental investigations, literature review and expert assessment is demonstrated for one variable.

1 INTRODUCTION

In many industrialised countries a pronounced deterioration of the condition and performance of engineering structures being observed. In the United States, for instance, more than 25% of bridges immediate repair or replacement [1]. Developing countries such as China are facing similar problems, with reports of average service lives of only 20 to 30 years for civil buildings and 10 to 20 years for marine ports [2, 3]. The main cause for the deterioration of concrete structures is the corrosion of steel reinforcement, with approximately two thirds of all defects in concrete being attributable to chloride-induced rebar corrosion [4, 5]. Premature onset of steel corrosion is in many cases caused by the ingress of deleterious agents such as chlorides into cracks of sometimes considerable width. These cracks may be avoided or their width reduced, if well-understood design principles are followed. However, the figures quoted above demonstrate that the practical application of said principles does often not result in sufficiently durable structures. Hence, there is an urgent need to improve the durability of cementitious materials on the material level.

To this end, new such as strain-hardening cement-based composites (SHCC) are being developed. SHCC are a group of high-performance fibre-reinforced cement-based composites characterised by their pseudo strain-hardening tensile behaviour (cf. Figure 1 and Table 1) [6, 7]. This is achieved by the formation of multiple cracks of self-limiting widths typically well below those observed in ordinary
reinforced concrete (RC) members. Thus, SHCC addresses the cause for the formation of wide cracks and subsequent premature steel corrosion on the material level. Details on the superior ductility and promising durability properties of SHCC and steel-reinforced SHCC (R/SHCC) members can be found in references [6-11].

![Typical stress-strain response and stress-mean crack width development for SHCC, cf. [12, 13].](image)

It can be shown in laboratory experiments, that the performance of R/SHCC members exposed to chlorides is indeed superior to that of RC members [14, 15]. However, it is unclear by how much and with what reliability the use of SHCC will increase durability and service life of real members and structures. Such information is required to justify the application of SHCC economically as well as with regards to sustainability, and it can only be obtained with performance-based durability and service life design concepts.

**Table 1 : Some SHCC and ordinary fly ash (FA) concrete compositions.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>335</td>
<td>331</td>
<td>193</td>
</tr>
<tr>
<td>Portland Cement</td>
<td>321</td>
<td>570</td>
<td>260</td>
</tr>
<tr>
<td>Fly Ash</td>
<td>749</td>
<td>684</td>
<td>140</td>
</tr>
<tr>
<td>Silica Fume</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Slag</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Coarse aggregates</td>
<td>-</td>
<td>-</td>
<td>949</td>
</tr>
<tr>
<td>Fine Sand</td>
<td>535</td>
<td>455</td>
<td>911</td>
</tr>
<tr>
<td>Super Plasticizer</td>
<td>16.6</td>
<td>5.1</td>
<td>4.8</td>
</tr>
<tr>
<td>Viscosity Agent</td>
<td>3.2</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PVA Fibre</td>
<td>29.3</td>
<td>26.0</td>
<td>-</td>
</tr>
<tr>
<td>w/c</td>
<td>1.04</td>
<td>0.58</td>
<td>0.74</td>
</tr>
<tr>
<td>w/b</td>
<td>0.78</td>
<td>0.44</td>
<td>0.61</td>
</tr>
</tbody>
</table>

\(^1\) Ratio of hydraulically active binder content \(w_{\text{binder}} = w_{\text{cement}} + 0.4 \cdot w_{\text{fly ash}} \leq 1.33 w_{\text{cement}}\) and \(w_{\text{binder}} = w_{\text{cement}} + w_{\text{silica fume}} \leq 1.11 w_{\text{cement}}\) as defined in DIN 1045-2 [18].

In this paper a performance-based durability concept for the initiation stage of chloride-induced steel reinforcement corrosion utilising fuzzy-probability theory is presented. This corrosion process is of significant importance since structural SHCC members are expected to contain steel reinforcement [9]. Further to that, the presented concept may serve as a template for the development of a compre-
hensive durability design framework for SHCC.

2 DURABILITY DESIGN STRATEGIES

To ensure the durability of an R/SHCC member, the exposure may either be avoided, or the member must be designed to withstand the environmental load. To this end, the following design strategies have been developed (cf. Figure 2):

- Deemed-to-satisfy rules prescribing e.g. a maximum w/b commonly specified in design codes for ordinary concrete [18-20].
- Durability indicators prescribing deemed-to-satisfy limits for properties such as the chloride diffusion coefficient, which can be determined in the lab and are directly linked to the corrosion process (e.g. [21]).
- Partial factor design, a semi-probabilistic approach typical for structural design with characteristic input variables and partial safety factors derived from probabilistic analyses (e.g. [22]).

Probabilistic design concepts such as the DuraCrete framework [22-24]. In these design concepts the deterioration processes is described by simple analytical equations, and the input variables are stochastically quantified to account for their variability. Using a limit state approach, this allows the computation of a member’s time-dependent failure probability and thus its durability and expected service life. Of these, only probabilistic design concepts a rigorous durability assessment and an economical service life design, which ensures that the desired service life is reached with the required reliability. However, the application of existing design concepts is limited to crack-free ordinary concrete. They cannot be applied to SHCC since they neither account for multiple finely spaced cracks characteristic of the material under service loads, nor for a composition that is very different from ordinary concrete.

Despite these shortcomings, the DuraCrete framework provides a rational basis for the development of a performance-based durability concept for SHCC. Its deterioration model must be extended to account for the contribution of cracks to the considered deterioration process, in this case chloride ingress. Further to that, the limited availability of data on SHCC deterioration precludes a purely stochastic quantification of input variables. For this reason, the uncertainty model of the new concept must allow for the transparent inclusion of additional sources of information such as expert knowledge, subjective assessment, and the transfer of experience and knowledge available for ordinary concrete. As will be seen, this can be achieved with the fuzzy-probabilistic design concept presented in this paper.
3 LIMIT STATE AND DETERIORATION MODEL

3.1 Limit State

Probabilistic design concepts require a limit state, which must be satisfied with a prescribed reliability throughout the service life. In the case of chloride-induced reinforcement corrosion, a common limit state is the end of the initiation phase at which a critical chloride concentration \( C_{\text{crit}} \) at the depth of the reinforcement is reached and the steel is depassivated. This yields the limit state function

\[
g(t) = C_{\text{crit}} - C(x = c, t).
\]

Failure occurs if the critical chloride concentration is exceeded. The probability of this occurring is described by Inequality 2:

\[
p_r = P\{g(t) < 0\} < p_{r,\text{max}}.
\]

3.2 Deterioration Model

Numerous models have been developed to describe chloride ingress into cementitious materials. Of these, empirical models are the most viable models for practical applications. However, their usefulness is limited by the need of very large data bases to quantify input variables that have no clear physical or (electro)chemical meaning [26-28].

One of the most common empirical models is Crank’s solution to Fick’s second law [29]. This was modified in the DuraCrete project and later by Gehlen and others [22, 24, 30] to account for differences between the boundary conditions assumed by Crank and field applications. It allows the calculation of the time-dependent chloride concentration \( C \) at the depth \( x = c \) of the steel reinforcement according to Equation 3:
with \( C_{e,0,ct} \): the constant apparent chloride concentration at the depth \( x \) of a zone in which the transport of chloride ions is dominated by convection, \( C_{0} \): the initial chloride concentration in the material, \( \text{erfc} \): the complement to the error function, and \( D_{s}(t) \) an apparent diffusion coefficient which is constant over the time of exposure \( t \) but different for different \( t \). Further modifications to the equation for the apparent diffusion coefficient allow a mathematically correct modelling of its time-dependence and to account for the contribution of cracks to the ingress of chlorides [31, 32]:

\[
D_{s}(t) = D_{ex, 0, cf} \cdot T_{cf} + D_{ex, 0, cr} \cdot T_{cr}
\]

with \( D_{ex, 0, cf} \) and \( D_{ex, 0, cr} \) the diffusion coefficients of the crack free material and the contribution of cracks determined at the reference time \( t_{0}^{cf} = 28 \text{ d under lab conditions.} \) The influence of environmental factors and the time-dependence of \( D_{s}(t) \), is described by

\[
T_{cf} = k_{e} \cdot k_{1} \cdot k_{2} \cdot \frac{1}{1-n_{cf}} \left[ \left( 1 + \frac{t_{ex}^{cf}}{t} \right)^{1-n_{cf}} - \left( \frac{t_{ex}^{cf}}{t} \right)^{1-n_{cf}} \right]^{n_{cf}}
\]

\[
T_{cr} = k_{e} \cdot k_{1} \cdot k_{2} \cdot \frac{1}{1-n_{cr}} \left[ \left( 1 + \frac{t_{ex}^{cr}}{t} \right)^{1-n_{cr}} - \left( \frac{t_{ex}^{cr}}{t} \right)^{1-n_{cr}} \right]^{n_{cr}}
\]

\[
k_{e} = \exp \left( b_{e} \left( \frac{1}{T_{ref}} - \frac{1}{T_{ref}} \right) \right)
\]

with \( k_{i} \) accounting for the influence of the test method used to determine \( D_{ex, 0, c} \). \( k_{e} \) accounting for the curing conditions, the age factors \( n_{cf} \) and \( n_{cr} \) describing the decrease of the apparent diffusion coefficient over time in crack-free and cracked material, \( t_{ex}^{cf} \) the age at first exposure, \( t^{cf} \) the time of exposure, \( b_{e} \) a regression parameter, the reference temperature \( T_{ref} = 293 \text{ K, and the actual temperature } T_{ref}. \)

4 Uncertainty Model

The uncertainty associated with input variables must be considered in the design to quantify the durability of SHCC members. Due to their different nature is useful to distinguish between stochastic and epistemic uncertainty. Stochastic uncertainty is a result of the inherent nondeterministic nature of a variable; it is objectively assessable and cannot be reduced. Imprecision or epistemic uncertainty, on the other hand, is the result of a lack of information. This may be due to an insufficient number and/or imprecise measurements. The resulting uncertainty cannot be eliminated, but it may be reduced, for instance by collecting more and more precise data, and through an improved understanding of the physical and (electro)chemical mechanisms of chloride binding [33].

To differentiate between both types of uncertainty, and to facilitate a transparent quantification of epistemic uncertainty, it is helpful to use different models in their description. Stochastics is well suited to describe randomness. Imprecision, on the other hand, may be described much more transparently using fuzzy set theory. Using so-called membership functions \( \mu(f) \), it allows a gradual assessment of the possibility that a variable attains a certain value on a scale from 0 to 1. No further limitations apply to the subjective quantification of these functions, but typically simple trapezoidal or triangular shapes (cf. Figure 3) are well suited to represent the available information.
Figure 3: Trapezoidal membership function $\mu_{\tilde{x}}(x)$ of the variable $\tilde{x}=\{x_1, x_2, x_3, x_4\}$, with the tilde denoting the fuzziness of the variable.

Since both types of uncertainty exist simultaneously, it is advantageous to describe the two phenomena in a fuzzy-random quantification of input variables [33, 34] (cf. Figure 4).

Figure 4: Fuzzy probability distribution function $\tilde{F}(x)$ for a normally distributed variable $\tilde{x}$ with a fuzzy mean value $\tilde{m}$ as defined by the fuzzy variable $\tilde{x}$ in Figure 3.

5 INPUT VARIABLE QUANTIFICATION

In Equation (3) the environmental load is reflected by the chloride concentration $\tilde{C}_{S,\text{ax}}$. Due to a small data base even for ordinary concrete, it is not uncommon to use expert judgement in its probabilistic quantification for ordinary concrete [24]. A fuzzy uncertainty model allows a more transparent variable quantification.

In case of permanent submersion in seawater, $\tilde{C}_{S,\text{ax}}$ becomes the surface chloride concentration. Assumed constant in Equation (3), it can be determined based on the mixture composition, the binding capacity of the matrix, and the seawater salinity according to Equation (8) [35]

$$C_{S,\text{ax}} = C_{\text{bound}} + V_{\text{pore}} \cdot C_{\text{Cl}}$$

with $C_{\text{Cl}}$ [g/l] the chloride concentration in the solution and $V_{\text{pore}}$ [1] the pore volume fraction. The concentration of bound chlorides $C_{\text{bound}}$ [kg/m³] is calculated by determining the hardened cement paste fraction of SHCC and multiplying it with the Freundlich binding isotherm

$$C_{\text{bound}} = \left(1 + W_0^b f_c \alpha h \right) \cdot (f_c C)^\beta$$

In Equation (8), it is $W_0^b = 0.25$, the weight of chemically bound water for each unit weight of hydraulically active binder and full hydration according to Powers, $f_c = w_{\text{binder}}/(w_{\text{binder}} + w_{\text{aggregates}})$, the frac-
tion of binder relative to the solids in the composition and $\alpha_h$, the degree of hydration. The parameters $\beta_m$ and $\beta$ of the Freundlich isotherm are tabulated in [35] for Portland and blended cements. For a blended cement with 30% fly ash, which is approximately equal to the maximum hydraulically active fly ash fraction [18], it is $\beta_m = 5.77$ and $\beta = 0.29$.

With this information, and conservatively assuming a degree of hydration of $\alpha_h = 1.0$, the surface chloride concentration can be calculated as $C_{S,ax} = 2.91\%$ by mass of binder. It is reasonable to assume, that this is the value $C_{S,ax}$ will most likely attain. Instead of making an assumption – typically without more than a cursory explanation – about a stochastic distribution, fuzzy probability theory allows the transparent quantification of an upper and lower bound for the variable. Together, these three values describe the triangular membership function of the variable similar to Figure 3.

As for the lower bound, the surface chloride concentration cannot be lower than for a matrix without binding capacity, in which only the pores are filled with the saline solution. According to Equation (7), this yields a lower bound of 1.30% by mass of binder. The upper bound of the fuzzy value for the surface chloride concentration can be determined based on experiments in which specimens were exposed to saline solution with $C_O = 165$ g/l [32]. To avoid being over conservative, the experimentally derived value is adjusted to account for a salinity of the pore solution of 20g/l instead of 165g/l. This results in an upper bound of the surface chloride concentration of 10.05% by mass of binder.

As a result, the surface chloride concentration may be quantified as

$$\hat{C}_{S,ax} = (1.30; 2.91; 10.05)\% \text{ by mass of binder} \quad (9)$$

All other input variables may be quantified analogously [36].

6 RESULTS AND THEIR INTERPRETATION

For fuzzy-random input variables the failure probability according to Inequality 2 will also exhibit fuzzy uncertainty. As can be seen in Figure 4b, this may lead to three different cases. If assessed conservatively, partial fulfillment is interpreted as failure.

To assess the sensitivity of results such as the chloride concentration $\hat{C}(c,t)$ to changes of the input variables, the coefficients of correlation between input variables $\hat{x}_i$ and the resulting chloride concentration $\rho_{C(c,t)x_i}$ were computed. These must be interpreted with some caution since they assume fuzzy values for fuzzy input variables and only yield linear correlations. Nonetheless, they help to identify of those variables, where a reduction of uncertainty has the most pronounced influence on reducing the uncertainty of results. This allows targeted future research [36, 37].

Figure 5: (a) Fuzzy probability density functions for load $S$, resistance $R$, and reliability $g$ and failure probability $p_f$; (b) Possible cases for reliability evaluation with fuzzy failure probability, from left to right (i) fulfilled, (ii) partly fulfilled, and (iii) not fulfilled.
7 CONCLUSIONS AND OUTLOOK

Strain-hardening cement-based composites have superior ductility and promising durability properties. To quantify their superior durability compared to ordinary concrete and to ensure efficiently designed SHCC members that attain the desired service life with the required reliability a performance-based design approach is required.

In this paper a fuzzy-probabilistic durability design concept for R/SHCC exposed to chlorides was presented. It allows the transparent inclusion of additional sources of information such as expert knowledge, subjective assessment, and the transfer of experience and knowledge available for ordinary concrete.

Pronounced non-stochastic uncertainties, may well at present limit the value of any results. Nonetheless, the potential of the approach is obvious. Furthermore, sensitivity analyses allow the identification of the most critical input variables, whose uncertainty can then be reduced through targeted research. In light of its potential, it is envisaged that the concept will in the future be developed into a comprehensive durability design framework for SHCC, encompassing steel corrosion propagation, other loadcases such as carbonation, as well as loadcase combinations.

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