DETERMINATION OF FIBRE ORIENTATION FACTOR IN HIGH AND ULTRA-HIGH-PERFORMANCE FIBRE-REINFORCED SELF-COMPACTION CONCRETE

B.L. Karihaloo and S. Kulasegaram
Cardiff University, Cardiff, UK

Abstract

This paper will describe a procedure for simulating the distribution of fibres and their orientation during the flow of self-compacting concrete (SCC) containing short steel fibres using the smooth particle hydrodynamics (SPH) mesh-less approach. It will then introduce a fibre orientation factor consistent with these three-dimensional simulations and describe how it can be easily determined from image analysis data on cut sections of cured SCC members.

1. INTRODUCTION

The self-compacting fibre-reinforced concrete (SCFRC) has high tensile strength and toughness, and excellent long term durability, low carbon footprint and thus sustainability. The concrete structures produced from SCFRC will also acquire these desirable characteristics at a competitive design life cost, provided the fibres within them are evenly and favourably oriented to the direction of principal tensile loading. According to the FIB Model Code (2010) a partial safety factor associated with fibre orientation called orientation factor is used in the design of SCFRC structures to account for the lack of information available to correlate the strength of a structure with fibre distribution and orientation. If the fibre distribution and orientation can be controlled and predicted effectively, this partial safety factor can be chosen close to unity thus allowing the superior properties of SCFRC to be exploited to the full extent.

A fibre orientation factor \( \eta (0 \leq \eta \leq 1) \) was first proposed [1] as the ratio of the number of fibres actually counted in a given cross-section \( N \) to the average theoretical number of fibres in such a cross-section \( N_{th} \), assuming that all the fibres are perpendicular to the cut surface. The theoretical number of fibres was chosen equal to the area of all fibres in the cross section \( (A_c \ast V_f) \) divided by the area of one fibre \( A_f \), where \( A_c \) is the area of the cross-section and \( V_f \) is the volume fraction of fibres [2]. The smaller this factor, the larger is the average deviation of the fibres from the perpendicular direction to the cut surface.

There are two drawbacks of this definition of the fibre orientation factor. Firstly, the approximate method [2] proposed for the calculation of \( N_{th} \) overestimates the actual theoretical number of fibres in a cross section. Secondly, \( N \) does not reflect the actual
inclination of the fibres relative to the section, and thus does not reflect the effectiveness of a fibre; a fibre is most effective if it is perpendicular to the cross-section; it is completely ineffective if it lies in the section; a fibre inclined to the section is partially effective depending on its angle of inclination.

In this paper a procedure for simulating the distribution of fibres and their orientation during the flow of self-compacting concrete (SCC) containing short steel fibres using the smooth particle hydrodynamics (SPH) mesh-less approach. A fibre orientation factor consistent with these three-dimensional simulations is then introduced. It is showed how it can be easily determined from image analysis data on cut sections of cured SCC members.

2. FLOW SIMULATION OF SCC MIXES WITH FIBRES

To develop a rigorous computational framework, two self-compacting concrete mixes containing 0.5% and 2.5% by volume 30 mm long steel fibres with crimped ends, called respectively high and ultra-high-performance mixes, were used for benchmarking the computational tools. A three-dimensional Lagrangian particle-based method, the so-called smoothed particle hydrodynamics (SPH), was used to simulate the flow. The constitutive behaviour of non-Newtonian viscous fluid was described by a Bingham-type model. The basic equations solved in the SPH are the incompressible mass conservation and momentum equations, together with the constitutive relation (equations 1-3 below).

\[
\frac{1}{\rho} \frac{D\rho}{Dt} + \nabla \cdot \mathbf{v} = 0, \quad \frac{D\mathbf{v}}{Dt} = -\frac{1}{\rho} \nabla P + \frac{1}{\rho} \mathbf{g} + \frac{1}{\rho} \mathbf{\tau}, \quad \mathbf{\tau} = \eta \dot{\gamma} + \tau_0 \left(1 - e^{-m\gamma}\right)
\]

In equations (1-3) \( \rho, t, v, P, g \) and \( \mathbf{\tau} \) represent the fluid particle density, time, particle velocity, pressure, gravitational acceleration and shear stress, respectively, and \( \eta \) is the plastic viscosity of the mix and \( \tau_0 \) is its yield stress. The large number \( m = 10^5 \) has been chosen to smoothen the bilinear Bingham constitutive relation.

As fibre orientation in SCFRC flow is mainly dictated by the flow of the entire mix rather than the mass of the fibres, it is feasible to assume that the positions and orientations of fibres are largely controlled by the fluid particles surrounding them. Accordingly, the entire domain can be discretized into SPH particles and rigid slender bodies that represent fibres. These rigid slender bodies will have the same material and geometrical properties as that of the actual steel fibres used in SCFRC. The initial positions and orientations of fibres are randomly generated. Further, as the fibre volume fraction in SCFRC is dilute (< 2.5%), fibres can be assumed to behave like rigid slender rods which undergo only rigid body translation and rotation during the flow. Therefore, the interaction between SPH fluid particles in the vicinity of the fibres and the rigid fibres can be modelled by treating fibres as immersed boundaries. However, in papers [3, 4] the fibres were represented by two end particles connected by a rigid massless rod. The increase in viscosity of the SCFRC due to the presence of fibres was calculated by a micromechanical procedure [5] and included in the plastic viscosity \( \eta \) (equation 3). Thus in the flow simulation of SCFRC by SPH, the fibres are treated as markers but the distance between the end particles is always maintained equal to the fibre length.

The solution procedure uses prediction-correction fractional steps with the temporal velocity field integrated forward in time without enforcing incompressibility in the prediction step [3, 4]. The resulting temporal velocity field is used in the mass conservation equation to satisfy incompressibility through a pressure Poisson equation. The numerical simulations of
SCFRC flow in two different geometrical configurations were carried out; the slump cone and L-box. The simulated results agreed very well with the test results.

To investigate how the short steel fibres will distribute and orient themselves during the flow, the slump cone tests of two mixes with different fibre volume content were simulated. In these simulations, the total number of particles used was 23581. The short steel fibres were treated as explained above. The number of fibres in each of these mixes was calculated from their volume fraction (0.5% in Mix 1 and 2.5% in Mix 2) (see Table 1). It should be mentioned that in this paper the focus is on the reorientation of fibres during the slump flow test for both mixes 1 and 2. The distribution of coarse aggregates in Mix 1 during the flow has been simulated in [6] and it shows that this mix does indeed flow as a homogeneous mass without any segregation of coarse aggregates or fibres. The results at two times after lifting the cone are shown in Figures 1-2.

Table 1: Volume fraction of fibres in Mix 1 and Mix 2 and the number of fibre end particles

<table>
<thead>
<tr>
<th></th>
<th>Mix 1</th>
<th>Mix 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of particles</td>
<td>23581</td>
<td>23581</td>
</tr>
<tr>
<td>Volume fraction of fibres (%)</td>
<td>0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Number of fibres</td>
<td>118</td>
<td>590</td>
</tr>
<tr>
<td>Number of fibre end particles</td>
<td>236</td>
<td>1180</td>
</tr>
</tbody>
</table>

Figure 1: Simulated horizontal flow of Mix 1 after 0.2 sec and 3 sec in slump cone flow test
As mentioned above, two particles represent the ends of each fibre and these particles are tagged throughout the flow of the SCC and the distance between them is maintained equal to the fibre length. The coordinates of ends of each fibre, are therefore known during the flow. It is thus easy to track the orientation of each individual fibre represented by angles $\theta_i$ and $\phi_i$ related to the fibre end particle coordinates throughout the flow of SCC, where $\theta_i$ spans between 0 and 90°, and $\phi_i$ between 0 and 360°.

A probability density function (PDF) of the fibre orientations $f(\theta, \phi)$ at an arbitrary instant $t$ during the flow is introduced. The PDF satisfies 
\[ \int_0^{2\pi} \int_0^{\pi} f(\theta, \phi) \sin \theta \, d\theta \, d\phi = 1. \]

A statistical analysis of the fibre orientations at different times during the simulated flow was performed to estimate the best fit PDF. The Johnson SB distribution function was found to fit the simulated data the best from among 55 different distribution functions tried, including the Gamma, Gaussian, uniform, log-normal, Weibull distributions, etc.

\[ f(x) = \frac{\delta}{\lambda \sqrt{2\pi z(1-z)}} \exp\left(-\frac{1}{2}\left(y + \delta \ln\left(\frac{z}{1-z}\right)\right)^2\right); \quad z = \frac{x - \xi}{\lambda} \]

In equation (4) $x$ stands for $\theta$ or $\phi$. It represents the most probable fibre density function for each of the two uncorrelated variables $\theta$, $\phi$. Here, $\lambda > 0$ is a scale parameter and $\xi$ is a location parameter. $\lambda$ and $\delta$ are shape parameters of the PDF curve. The density function is skewed to the left, symmetric, or skewed to the right, if $\gamma > 0$, $\gamma = 0$, or $\gamma < 0$, respectively. $\delta$ represents the standardized measure of kurtosis of the curve; a high positive value means the curve is sharp, while a small value close to 0 means a nearly flat, wide curve. The mean values of angles $\theta$ and $\phi$ at various instants of flow shown in Figure 3 are calculated using the following equation, where $n$ is the total number of fibres.

\[ \bar{x} = \sum_{i=1}^{n} f(x_i) x_i \]

where $x$ stands for $\theta$ or $\phi$. 

Figure 2: Simulated horizontal flow of Mix 2 after 0.2 sec and 3sec in slump cone flow test
From Figure 3, it is clear that fibres align themselves with the flow such that the mean angle of orientation with respect to the horizontal direction of flow is only \((90-\theta) \approx 8^\circ\) after 10 s, irrespective of the volume fraction of fibres. This is in agreement with the probability density function \(f(\theta)\) which is skewed to the right. The fibre orientation is therefore mainly dictated by the fluid flow of the homogeneous SCC mix rather than the mass of fibres, and this confirms the assumption made in the SPH simulation that the positions of the particles representing a fibre are largely controlled by the fluid particles surrounding them. The fibres however remain randomly distributed in the circumferential direction as attested by the probability density function \(f(\phi)\) which remains nearly a uniform distribution throughout the flow with the mean \(\phi\) deviating from the anticipated \(180^\circ\) by less than \(6^\circ\) in Mix 1 (low volume fraction of fibres) and by less than \(3^\circ\) in Mix 2 (high volume fraction of fibres) (Figure 3) after 10 s.

The fibre orientations are simulated in a similar manner in L-box and used below to demonstrate the determination of the fibre orientation factor. The number of particles used for this simulation are given in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Mix 1</th>
<th>Mix 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of particles</td>
<td>59568</td>
<td>59568</td>
</tr>
<tr>
<td>Volume fraction of fibres (%)</td>
<td>0.5</td>
<td>2.5</td>
</tr>
<tr>
<td>Number of fibres</td>
<td>298</td>
<td>1490</td>
</tr>
<tr>
<td>Number of fibre end particles</td>
<td>596</td>
<td>2980</td>
</tr>
</tbody>
</table>

### 3. FIBRE ORIENTATION FACTOR

From the simulations of the orientations of the fibers during the flow it was possible to calculate the orientation factor envisaged in the *FIB Model Code (2010)*.

Figures 4-5 illustrate how the steel fibres are oriented in vertical sections of the self-compacting concrete mixes (Mix 1 and Mix 2) at one time interval during flow in the horizontal part of the L-box (the distance is measured from the gate). For clarity of presentation, the sizes of the ellipses, circles, and rectangles are exaggerated several times.
compared to the cross section dimensions of the L-box. Note that the coordinates of the particles representing the ends of the fibres are continuously monitored throughout the simulation from which the cross-sectional shapes of the fibres cut by the vertical sections are calculated.

It should be mentioned that the end particles representing the fibres are randomly distributed, but as the distance between these particles is fixed and equal to the fibre length, a number of degrees of freedom are suppressed. Referring to Table 2, this means that 298 degrees of freedom out of a possible 178,704 are suppressed in Mix 2 and 1,490 in Mix 4.

![Figure 4: Vertical cross sections at 200 mm (left) and 300 mm (right) after 3 s in L-box (Mix 1)](image1)

![Figure 5: Vertical cross sections at 200 mm (left) and 300 mm (right) after 2 s in L-box (Mix 2)](image2)

Figures similar to 4 and 5 at different times during the flow reveal that the number of fibres cut by the vertical section decreases as the flow time increases and the surface area of cut section reduces. It should be mentioned that for the L-box, when the flow stops, the surface area of all cut sections should be nearly the same, therefore the number of fibres should be
same in all the vertical cross sections. For a random distribution of fibres in a volume, the number of fibres lying in section of a certain area or cut by it can be calculated exactly using the theory of geometric probability, or more precisely from the solution of the so-called Buffon problem [8]. The number of fibres in the cut section in the simulations is compared with the theoretical value in Table 3.

Table 3: Number of fibres in a vertical cross-section of (Mix 1) and (Mix 2) based on the 3D simulation and Buffon problem

<table>
<thead>
<tr>
<th>Flow time (s)</th>
<th>No. of fibres in the simulation</th>
<th>No. of fibres based on Buffon problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section (mm)</td>
<td>200 300</td>
<td>200 300</td>
</tr>
<tr>
<td>Mix 1</td>
<td>2 s 13 8</td>
<td>13 10</td>
</tr>
<tr>
<td></td>
<td>3 s 16 11</td>
<td>13 11</td>
</tr>
<tr>
<td>Mix 2</td>
<td>2 s 63 51</td>
<td>62 49</td>
</tr>
<tr>
<td></td>
<td>3 s 64 58</td>
<td>62 56</td>
</tr>
</tbody>
</table>

Bearing in mind the drawbacks of the orientation factor first introduced in [1], a new orientation factor was introduced in [7]. It is defined as the ratio of the projected length along the normal to the cut plane to the actual length \( l \) of the fibre. This ratio is nothing but the cosine of the angle between the fibre and the vertical axis of the cut section. This definition is consistent with the fibre probability density function introduced in this work. The orientation factor in the vertical cut plane is:

\[
\eta = \frac{1}{n} \sum_{i=1}^{n} \cos \theta_i = \frac{1}{n} \sum_{i=1}^{n} \frac{r}{a_i} \tag{6}
\]

where \( n \) is the number of fibres in the cut section, and \( r \) is the radius of the fibre. It is clear from Figure 6 that the fibres tend to reorient themselves with the direction of flow, irrespective of the fibre volume fraction; corresponding to a fibre orientation factor of about 0.90 and 0.92, respectively.

By comparison, according to the definition of [1] the fibre orientation factor for Mix 1 and Mix 2 after 2 s flow of the L-box at 200 mm cross-section would be \((11/16) = 0.69\) and \((13/19) = 0.68\), respectively. The corresponding values after 3 s flow of the L-box at 300 mm cross-section for Mix 1 and Mix 2 are \((56/81) = 0.69\) and \((62/90) = 0.68\), respectively. In other words, the fibre orientation factor hardly changes with time, i.e. the fibres do not reorient with the principal flow direction at all which is clearly unexpected and counterintuitive in SCC mixes. In calculating the fibre orientation factor according to [1] we have used \( N_{th} \), as explained above and \( N \) according to the solution of the Buffon problem (Table 3).

Finally, it should be mentioned that in practice it may not be possible or practicable to perform the full three-dimensional simulation of the flow of fibre-reinforced SCC into the formwork and thus to determine the mean angle of inclination of fibres \( \theta \) needed for the calculation of the fibre orientation factor (first equality in Equation (6)). In such instances, it is necessary to count all the fibres in the cut section, noting the fibres with circular sections, measuring the major axis of all fibres with elliptical sections, and using the second equality in Equation (6) to calculate the fibre orientation factor. Note that the fibres that lie in the plane
along which the specimen is cut (i.e. $\theta_i = 90^\circ$), and which are thus likely to come loose and fall during cutting, need not be taken into account in any case because they make a negligible contribution to the fibre orientation factor.

![Graph showing fibre orientation factor in vertical cut section at 100 mm in the L-box (left Mix 1; right Mix 2)](image)

**REFERENCES**


