Abstract

Results from early-age material testing implies that the macroscopic uniaxial compressive strength of cementitious materials increases first overlinearly and then linearly with increasing hydration degree [1,2]. We here quantitatively explain these macroscopic observations based on one universal hydrate strength value which intervenes in an elasto-brittle strength law formulated for micron-sized needle-shaped hydration products (called hydrates), as well as on a continuum micromechanics model accounting for the characteristic microstructure of cementitious materials and allowing for upscaling the microscopic strength law to the macroscopic “material” scale of cementitious materials [3,4]. Model predictions agree well with results from macroscopic experiments on hydrating cement pastes and sprayed concretes, with different compositions and different hydration degrees. In addition, the hydrate strength value is consistent with nanoindentation-related hardness values of hydrates, which are by one order of magnitude larger than the hydrate strength [3].

1. INTRODUCTION

It is known from early-age material testing that the macroscopic uniaxial compressive strength of cementitious materials increases linearly with increasing hydration degree, once a critical degree of hydration has been surpassed [1,2]. We describe a model elucidating (i) the microstructural material characteristics which drive this dependence, as well as (ii) the nature of the hydration degree-strength relationship before the aforementioned critical degree of hydration is reached. The micromechanical model provides hydration degree-strength relationships of cement pastes [3] and sprayed concretes [4], covering a large range of compositions. Herein, we compare model predictions with results from early-age strength tests on several different cementitious materials, carried out at three different laboratories [1,5,6].

2. FUNDAMENTALS OF CONTINUUM MICROMECHANICS

Quasi-homogeneous material properties of microheterogeneous media are defined on representative volume elements (RVEs) which satisfy the separation of scales principle. This
implies that the characteristic size of the RVE is (i) significantly larger than the characteristic size of the heterogeneities, and (ii) significantly smaller than the characteristic length of stress and strain fluctuations within the structure containing the RVE.

Since the microstructure within RVEs of microheterogeneous materials cannot be described in complete detail, quasi-homogeneous subdomains (so-called material phases) are identified. Once their mechanical behavior, their dosages within the RVE, their characteristic shapes, and the mode of their interaction are known, the „homogenized“ mechanical behavior of the overall material can be estimated [7], i.e. the relation between homogeneous deformations acting on the boundary of the RVE and resulting (average) stresses, or the ultimate stresses sustainable by the RVE, respectively.

In hierarchically organized materials, a material phase, identified at a specific scale of observation “A”, may exhibit a heterogeneous microstructure on a lower scale of observation “B”. The mechanical behavior of this microheterogeneous phase can be estimated by that of an RVE with a characteristic size being smaller than or equal to the characteristic size of the aforementioned phase, i.e. that of inhomogeneities identified on observation scale “A”, see, e.g., [8].

3. CONTINUUM MICROMECHANICS OF CEMENTITIOUS MATERIALS

For cement paste, we employ two RVEs (Fig. 1): The first one relates to a polycrystalline hydrate foam with spherical phases representing water and air, and with needle-shaped phases of hydration products exhibiting isotropically distributed orientations, see Fig. 1(a) and [3]. The second one relates to cement paste with a spherical phase representing unhydrated cement clinker embedded in a continuous hydrate foam matrix, see Fig. 1(b). The RVE of sprayed concrete comprises a spherical phase representing sand grains and aggregates embedded in a continuous cement paste matrix, see Fig. 1(c).

Isotropic elasticity constants of clinker, water, hydrates, air, and aggregates are accessible from ultrasonics and nanoindentation testing, described in the open literature, see, e.g., [3] and references therein. Volume fractions of the material phases depend on the initial composition of cementitious materials as well as on their maturity. This is modeled by Powers and Acker’s hydration model [9,10] which accounts for the composition through the initial water-to-cement mass ratio \((w/c)\) as well as the aggregate-to-cement mass ratio \((a/c)\) and for the maturity through the hydration degree \(\xi\). The latter is equal to zero at the time instant of mixing the material, and it attains the value one once all available cement clinker is consumed by the chemical reaction.

As for modeling strength, we consider (i) that cementitious materials behave linear elastic as long as deviatoric stress peaks in the micron-sized needle-shaped hydrates remain below a related hydrate strength value and (ii) that reaching this hydrate strength in the most heavily stressed region of the hydrate phases corresponds to overall failure of the material [1,11,12]. The required macro-micro transition from the macrostrain imposed as boundary condition on the RVE to the deviatoric stress peaks in micron-sized needle-shaped hydrates with an arbitrary orientation in space is provided by so-called quadratic stress averages. They are derived from the elastic energy stored within the RVE [13,14], and they essentially involve the derivative of the fourth-order elasticity tensor with respect to the shear modulus of the oriented hydrates. Related differential quotients can be computed reliably by means of finite difference expressions [11,12]. The described strength approach can be interpreted as an
elastic limit-based von Mises criterion for brittle failure of micron-sized needle-shaped hydrates.

The order of magnitude of the hydrate strength is identified by setting the uniaxial compressive strength of a mature cement paste with initial water-to-cement mass ratio amounting to 50% equal to a corresponding model prediction for a fully cured cement paste with the same composition [3]. This implies that the order of magnitude of the hydrate strength amounts to 50 MPa.

![Image of hierarchical organization of cement paste and sprayed concrete](image)

4. COMPARISON OF MODEL PREDICTIONS WITH EXPERIMENTS

We evaluate the described micro-elasto-brittle model for six different cement pastes (with initial water-to-cement mass ratios ranging from 15.7% to 80%), and for hydration degrees ranging from zero up to almost completed hydration. These model predictions agree very well with experiments of Taplin [1]: the quadratic correlation coefficient amounts to 93%, the mean prediction error is equal to –3% and the standard deviation amounts to 7.3%. This implies that the identified hydrate strength is consistent with macroscopic strength increase of hydrating cement pastes [3].

Next, we evaluate the described micro-elasto-brittle model for two different sprayed concretes exhibiting maturities ranging from zero to 70%. The initial composition of the first material is defined through $w/c = 0.5$ and $a/c = 3.8$, while the second material exhibits $w/c = 0.4$ and $a/c = 3.94$. Model predictions agree very well with results from experiments on sprayed concretes with these two compositions, carried out in two different laboratories [5,6].
The quadratic correlation coefficient amounts to very satisfactory 98.6%. This implies that the identified hydrate strength is also consistent with macroscopic strength increase of hydrating sprayed concretes [4].

![Figure 2: Comparison of model predicted strength evolutions for cement pastes with different initial water-to-cement mass ratios [3] with experiments from Taplin [1]](image)

![Figure 3: Comparison of model predicted strength evolutions for sprayed concretes with different compositions in terms of initial water-to-cement mass ratio and aggregate-to-cement mass ratio [4] with experiments from Pillar [5] and Lafarge [6]](image)

Finally, we highlight that the hydrate strength (amounting to 50 MPa) is consistent with hardness values of low-density and high-density calcium-silicate-hydrates (amounting to 450 MPa and 830 MPa, respectively) obtained from nanoindentation testing [15]. To this end, we first inscribe a Tresca failure surface into the cylindrical von Mises surface defined by the failure criterion described in Section 3. This results in a cohesion value amounting to 43.3 MPa, which can be interpreted as a pure shear strength [3]. The latter is related – by means of
yield design approaches [16] – to an interval of hardness values (ranging from 272 MPa to 1704 MPa) when considering nanoindentation with a Berkovich tip and friction angles of hydrates between 3° and 30°, see Fig. 6 of [16]. This interval contains the above-reported nanoindentation measurements, which implies that the much smaller hydrate strength value is also consistent with nanomechanical investigations on cementitious materials [3].

5. CONCLUSIONS

One « universal » hydrate strength, representing the basis for elasto-brittle strength upscaling for cementitious materials allows for prediction of the macroscopic strength evolutions of hydrating cement pastes and sprayed concretes, with different compositions. In addition the hydrate strength value is consistent with results from nanomechanical characterization of hydration products, in the framework of grid indentation techniques.

REFERENCES
