A FILAMENT BEAM-COLUMN MODEL FOR THE NON-LINEAR ANALYSIS OF RC FRAMES INCLUDING THE EARLY AGE EFFECTS

Antonio R. Marí (1), María D. Crespo (2), Climent Molins (1), Jesús M. Bairán (1) and Denise Ferreira (1)

(1) Department of Construction Engineering, School of Civil Engineering of Barcelona, Universitat Politècnica de Catalunya, Spain

(2) Department of Structures, School of Civil Engineering, Rosario National University, Argentina

Abstract
A fiber beam model able to reproduce the strain and stress state of tridimensional reinforced and prestressed concrete frame structures since early ages is presented. The model is based on the Finite Element Method accordingly to the Timoshenko beam theory. The cross-section is discretized into concrete fibers and longitudinal steel filaments, allowing the consideration of different levels of maturity for each fiber. Transversal steel is considered smeared in the concrete fibers. Strains, in each instant of time, are given by the summation of different sources, such as, thermal strains, shrinkage, creep and aging. In this paper two application examples are shown: i) the study of the early age behavior of concrete elements with total external restraint and ii) the numerical simulation of the temperature and strain/stress development in a large beam specimen loaded in bending after free shrinkage.

Résumé
On présente un modèle numérique pour l’analyse non linéaire de structures basé sur un élément fini unidimensionnel de Timoshenko capable de reproduire l’état de contraintes et de déformations des ossatures en béton armé ou précontraint au jeune âge. La section transversale est composée de fibres en béton ou en acier, permettant de reproduire le degré de maturité propre à chaque fibre. Le ferraillage transversal est considéré comme réparti sur les fibres de béton. Les déformations considérées sont celles provenant de la température, du fluage, du retrait et du vieillissement du béton. Dans cet article, on présente deux exemples d’application du modèle développé: i) l’étude du comportement au jeune âge d’éléments totalement bloqués à leurs extrémités et ii) la simulation numérique des températures et du développement des contraintes et déformations dans une grande poutre chargée en flexion après avoir subi une phase de libre retrait du béton.
1. INTRODUCTION

Early age stresses arise from the restraint to the volumetric changes (thermal strains and autogenous shrinkage strains) that develop from the hydration process of the cement. In order to study the structural effects of the phenomena that take place at early ages, analytical models capable to take into account the non-linear and time dependent response of the structures at early ages and along the structure service life are necessary. Although the volumetric strains at early ages produce 3D effects, in case of beams and columns, it can be considered that the moisture and thermal flow produced are essentially longitudinal, except at the elements ends. In addition, recently developed beam-column models may reproduce also with good accuracy the response of frame structures under shear and normal forces, thus allowing a good simulation of the structure behavior under loads and imposed deformations. Since the temperature distribution is nonlinear within a cross section, the consequent strain field is also nonlinear. In case of structures with no external restraint, these nonlinear strains induce self equilibrated stresses but when the external restraint is no null, internal forces are also produced. In this paper, a nonlinear numerical model able to simulate the strain/stress state of reinforced concrete (RC) elements since early ages based on the fiber beam element formulation is presented [1-2]. Afterwards, the developed computational tool is applied to the study of two structures with fairly diverse structural behavior due to theirs different degree of external restraint: i) an element with total external restraint and ii) a large beam specimen loaded in bending after free shrinkage.

2. NUMERICAL MODEL

2.1 Modelling the structural response at mature age

In order to reproduce the non-linear and time-dependent behaviour of frame structures under loads and imposed deformations, such as creep, shrinkage and thermal effects, a filament 3D beam-column model was developed by Mari [3-4]. The model takes into account the dominant flexural behaviour, as previous models [5-8]. In the present work, an improved beam-column model that incorporates the interaction between shear and normal stresses, Ferreira et al. [1] has been used. In addition, the model has been adapted to take into account the thermal and rheological effects that occur in concrete at early ages [2].

The model is based on the displacement formulation of the Finite Element Method (FEM), using a Timoshenko beam element with 13 DOF (6 at each node plus one internal DOF). The cross section is divided into fibers or filaments of concrete and steel.

The concrete is assumed to be subjected to a biaxial stress state. The Hognestad parabola [9] with descending post-peak linear branch is taken as the backbone curve for concrete in uniaxial compression. According to the Modified Compression Field Theory [10] compression softening is taken into account through a factor $\beta$ for the compression-tension state and the strength enhancement factor $K_c$ [11] is considered in biaxial compression. A smeared crack model with rotating cracks is used and tension stiffening, adopting Cervenka’s model [12], is considered in the tensile stress-strain branch of concrete. The evolution of the concrete mechanical properties due to aging with time has been considered according to the Eurocode EC-2. For the reinforcing steel, a bilinear stress-strain relationship is assumed with load reversals. Accordingly to the section state determination, mixed strategies are used to simulate the shear resistant mechanism of cracked reinforced concrete. On one hand, plane
section assumption is assumed to determine longitudinal strains and rotations and, on the other hand, a force-based approach assigns a shear stress pattern to the cross-section. The total strain at a given time and point in the structure $\varepsilon(t)$, is taken as the direct sum of mechanical strain $\varepsilon^m(t)$, and non-mechanical strain $\varepsilon^{nm}(t)$, consisting of creep strain $\varepsilon_{cr}(t)$, shrinkage strain $\varepsilon_{sh}(t)$, aging strain $\varepsilon_{a}(t)$, and thermal strain $\varepsilon_{T}(t)$.

$$\varepsilon(t) = \varepsilon^m(t) + \varepsilon^{nm}(t)$$

(1)

$$\varepsilon^{nm}(t) = \varepsilon_{cr}(t) + \varepsilon_{sh}(t) + \varepsilon_{a}(t) + \varepsilon_{T}(t)$$

(2)

Creep strain of concrete is evaluated by an age dependent integral formulation based on the principle of superposition. The shear-normal stresses interaction is taken into account at the sectional level by adopting a constant shear stress along the cross section depth. In addition to typical equilibrium equations between the cross section stresses and internal forces, vertical equilibrium between the forces in the transverse reinforcement and the vertical stresses in concrete is set at each filament level. Through the use of a set of equilibrium, compatibility and constitutive equations, a relationship between the normal and shear internal forces and section deformations (axial strains, curvatures and shear angular strain) is obtained in a matrix form that is in general full, showing coupling between shear and normal behavior.

The structural analysis strategy consists of a time step-by-step procedure, in which the time domain is divided into a discrete number of time intervals. A time step forward integration is performed in which increments of displacements, strains and other structural quantities are successively added to the previous totals as forward in the time domain. At each time step, the structure is analyzed under the external applied loads and under the imposed deformations, such as creep, originated during the previous time interval and geometry.

Nodal displacements, element internal forces, stresses and strains at each concrete and steel filament, curvature and elongation of each section, support reactions and other response parameters are provided by the model, after convergence is reached. The described model was experimentally checked by Marí and Valdés [13].

### 2.2 Modelling of the structural response at early ages.

As the cement hydration process is thermo-activated and as the volumetric changes generate stresses in the case of being restrained, there are interactions between the cement hydration, thermal and mechanical problems. As the interaction between the mechanical and the thermal and the hydration problems is depreciable, the numerical treatment of these interactions gets fairly simpler. In this manner, the problem of the cement hydration can be solved with the thermal problem and afterwards the mechanical one can be taken separately.

If the validity of the Fourier’s law is accepted, the field equation for the thermal problem is given by:

$$CT - Q - k \nabla \cdot (\nabla T) = 0$$

(3)

where $T$ is the temperature, $C$ is the heat capacity per unit of volume, $Q$ is the heat generation rate per unit of volume and $k$ is the thermal conductivity.

The boundary conditions are taken as a convective flow:

$$q_n = h (T_{ext} - T)$$

(4)
where $q_n = -k \frac{\partial T}{\partial n}$ is the heat flow normal to the border of the structure, $h$ is the total thermal transfer coefficient and $T_{\text{ext}}$ is the external ambient temperature. The heat released by the hydration process up to the instant $t$ is given by:

$$Q(t) = \alpha(t) Q^{\text{max}}$$  \hspace{1cm} (5)

where $\alpha(t)$ is the degree of hydration and $Q^{\text{max}}$ is the maximum heat of hydration, given by the summation of the maximum heats of the hydration of different reactive phases of the cement accordingly to the procedure proposed by Bogue. In this work, the degree of hydration is expressed as a function of the equivalent age, fitted in accordance to the three-parameter model of Freisleben-Hansen and Pedersen [14].

The heat transmission equation (3), along with the boundary conditions (4) and the initial temperature conditions has been solved by the use of the Finite Element Method and implemented in the numerical program Hydra-Analysis [2], developed from the numerical program Fire-Analysis [15], that allow 2D analysis with 8-noded isoparametric finite elements.

The mechanical problem is solved through the numerical program Early-CONS [2]. In this model, the instantaneous strains, the autogenous shrinkage, creep and the development of the mechanical properties can be simulated as functions of the equivalent age or of the degree of hydration. A comprehensive description of the implemented models can be found in [2] where a good fitting with the different phenomena is also demonstrated.

3. APPLICATION EXAMPLES

3.2 Elements with total external restraint

Some Codes limit the crack widths generated by imposed constrained strains by setting a minimum reinforcement ratio determined as function of the normal forces that arise in the member at the cracking moment in a simplified tie model, preventing the yield of the steel. As the external restraint goes along with the internal restraint, the influence of the former in the crack formation is of great interest, even though being a topic of discussion [16].

The goal of this study is to analyze the influence of the concrete, formwork, height of the section and the age of removing the formwork in the development of normal forces in the reinforced concrete elements with total strain restraint in order to draw conclusion about the values used to determine the minimum reinforcement ratio.

Table 1: Concrete properties

<table>
<thead>
<tr>
<th>Property</th>
<th>CO1</th>
<th>HAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ck}$ [MPa]</td>
<td>25</td>
<td>52</td>
</tr>
<tr>
<td>Cement [kg/m$^3$]</td>
<td>270 (CEM I 32.5R)</td>
<td>425 (CEM I 52.5R)</td>
</tr>
<tr>
<td>Fly ashes [kg/m$^3$]</td>
<td>60</td>
<td>-</td>
</tr>
<tr>
<td>Calcareous filler [kg/m$^3$]</td>
<td>-</td>
<td>22.3</td>
</tr>
<tr>
<td>Water [kg/m$^3$]</td>
<td>175</td>
<td>180</td>
</tr>
<tr>
<td>$Q^{\text{max}}$ [kWh/m$^3$]</td>
<td>39.0</td>
<td>54.4</td>
</tr>
<tr>
<td>Autogenous shrinkage $[\mu\varepsilon]$</td>
<td>-</td>
<td>223</td>
</tr>
</tbody>
</table>
Elements of different thickness built with two concretes were analyzed. Concrete CO1 was characterized in the iBMB (Institut für Baustoffe, Massivbau und Brandschutz, Technische Universität Braunschweig) [17], and the self-compacting concrete HAC was characterized through experimental tests carried out in the LTE-UPC (Laboratorio de Tecnología de Estructuras de la Universidad Politécnica de Catalunya) [2]. The main properties of both concretes are presented in Table 1. The convection coefficients used represent the effect of a plywood formwork with 18 mm thick and a metallic formwork up to 28 mm thick in standard environmental conditions.

Only some results of the mechanical study related to an element with a cross section 80 cm-high, cast with an initial and environment temperature of 20°C and with formwork removal at two days of age are presented in the following. The variation of normal stresses are shown in the Figure 1: for the case of the concrete C01 the cross section does not crack, in contrast, for HAC the cross section is fully cracked before removing the formwork, with an earlier and quicker development of cracks for the case of metallic formwork.

![Figure 1: Development of normal stresses in a cross section: dc=80 cm - Ti=Ta=20°C - td=2d](image1)

![Figure 2: Development of the stress profiles. HAC - dc=80cm – Ta=Ti=20°C – td=2d](image2)
The development of the stress profiles for the HAC are presented in the Figure 2: the compressive stresses at the beginning of the hydration process are more uniform and reach higher values for the case of the plywood formwork; therefore, the cross section starts to be under tensile stresses in a later stage in comparison with the metallic formwork. At the age of 36 hours, the cross section is still in compression; the removal of the formwork generates an abrupt increase in the bound tensile stresses and shortly after the 49 hours of age the cross section starts to crack from the exterior fibers. In contrast, for the case of the metallic formwork, at around 43 hours of age (before removing the formwork) some fibers at the center of the cross section start to crack.

In relation to the CO1 concrete, the stress development is similar for the generality of the cross sections, in which, in spite of the fact that cracking is not reached, at the end, tensile stresses take values that are quite similar to the concrete strength. Accordingly to the plywood formwork, the stress profile is rather uniform at the beginning and at the end of the process. Few hours after removing the formwork, the stresses increase abruptly in the external part of the cross section originating an important stress gradient. Alternatively, removing the metallic formwork does not significantly modify the stress profile.

3.3 Large beam specimen

CEOS.fr (http://www.ceosfr.org/) has carried out an experimental campaign for benchmark purposes (ConCrack, http://www.concrack.org/) in order to evaluate the simulation skills of the models to describe the cracking phenomenon of RC structures. One of the experimental tests consisted in a large RC beam specimen loaded in bending after free shrinkage. In order to simulate the cracking propagations process of the beam while being under load, the estimation of early age strains and stresses caused by the temperatures developed by the hydration process of cement and autogenous shrinkage were taken into account.

![Figure 3: Photo of the beam specimen before being placed in the testing bench](image)

The beam specimen which was cast with a 50 MPa characteristic compressive strength concrete, with 6.10 m long, 1.6 m wide and 0.8 m high was placed above wooden ties until the moment it was carried to the testing bench (Figure 3). Both lateral and the bottom faces of the beam were in contact with a 2 cm width wood formwork, which was removed from the lateral faces at two days of age.

Since the beam length was much higher than the dimensions of the cross section, the heat flow in the longitudinal direction was depreciated and a 2D finite element analysis with 8-
noded isoparametric elements was carried out with the Hydra-Analysis program. The cross section was discretized into 800 elements, 20 equal length divisions in the horizontal direction and 40 equal length divisions in the vertical directions. Time domain was discretized into steps of 15 minutes in the first 6 days and afterwards of 1 hour duration for the remaining days in the case of the prediction and of 15 minutes in the *a posteriori* simulation.

The thermal characterization of the concrete was based on its dosage, chemical composition of the cement and on one adiabatic test, which started 3 hours after the placing of concrete due to operative questions. The final rate of hydration based on the temperatures measured in the adiabatic tests was of 0.61, which is a very low value for concretes with a water/cement rate as the one used (0.46); that might be due to the late start of the adiabatic test. For this reason, the development of the degree of hydration was modeled according to Freiesleben-Hansen y Pedersen [13], using the parameters proposed by [19] that are taken in function of the percentages of the reactive phases.

![Figure 4: Measured and computed temperatures from the prediction simulations](image)

In the Figure 4, measured temperatures are compared with the results predicted by the numerical simulations carried out by the different research groups that participated in the benchmark [20]. The results depicted in magenta color, identified as T31, belong to the predictions obtained by the authors. For the superior and central monitored points (SL and C), it can be observed that the computed temperatures that better approximate the experimental values are the ones represented in magenta color (T31) and cyan blue (T8). However the prediction of T8 team shows a slower development when compared with the ones computed by team T31 and also with the measured ones. In relation to the inferior monitored point (SL), all the predicted temperatures, with the exception of team T31, were lower than the experimental ones. Temperatures obtained in the prediction simulation are in good accordance
with the experimental measures, taking values slightly greater than the sensor located in the superior and central areas (Su and C) and also slightly inferior to the measurements of the sensor located in the inferior area (SL). In relation to the kinetics progress, numerical temperatures presented a faster development when compared to the measurements.

In order to better adjust the numerical results to the measured temperatures, an *a posteriori* simulation was carried out in which the parameters of the degree of hydration curve were slightly modified and also a convection coefficient in the inferior face of the beam was taken as 2 instead of 4 [W/m²°C] previously used, reaching a better fitting to the experimental data.

![Figure 5: Stress state at the beginning of the loading test](image)

The early age mechanical analysis was carried out considering a linear constitutive equation. As the beam had no longitudinal restraint, the stress state in the cross section becomes self balanced, with tensile stresses in the outline area and compressive stresses in the center in the heating phase. The development of the elasticity modulus causes a sign stress reversion in the cooling phase with compression stresses in the outline area and tensile stresses in the center of the cross section (Figure 5).

The static load was applied in the beam during the mechanical test through two parallel jack rows until reaching a total load of 2250 kN per row. The mesh used in the mechanical simulation of the loading test consisted in 36 finite elements in the longitudinal direction and

![Figure 6: Maximum vertical displacements](image)

(a) Pre and postdiction results

(b) Comparison with other participants
the cross section discretization was the same as the one used in early age thermo-mechanical analysis. Maximum vertical displacements obtained by the a priori and a posteriori simulations are compared with the experimental measurements in the Figure 6a), and in the Figure 6b) a comparison of numerical results obtained by other participants is shown. A general good fitting can be observed.

In Figure 7, computed displacements at midspan are depicted for the analysis taking into account the stress state generated by the early age thermo-mechanical problem (EA+ LOAD) and the analysis that does not take this into account (LOAD). When early age stresses are taken into account, the stiffness loss due to cracking starts for lower loads and in a more pronounced manner, leading to a maximum displacement of approximately 19% greater than the one determined with the pure mechanical analysis.

![Graph](image)

Figure 7: Displacements at mid-span considering or not early age effects

4. CONCLUSIONS

The computational tool presented allows the simulation of the crack formation process of RC linear elements since early ages. The stress profiles of elements with total external restraint are strongly dependent on the different intervenient variables and cracking does not happen in an instantaneous manner, so the determination of the normal forces to obtain the minimum reinforcement ratio in order to avoid large cracks is of rather complex definition. In relation to the large beam specimen a very good correlation was obtained between measured temperatures and the ones obtained numerically with the chosen theoretical curve for the development of the degree of hydration. The early age behavior influences the latter structural response of the beam.

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REFERENCES


