Abstract
The objective of this study is the characterisation and modelling of coupling between drying shrinkage and damage in concrete material. In the first part, we present an experimental study on a classical concrete in order to characterise damage induced by drying shrinkage. Uniaxial compression tests are realised at different ages of concrete. The decrease of Young’s modulus is related to the mass loss of specimen. These results are commented in terms of transport properties of material. In the second part, a constitutive model for partially saturated concrete is proposed. It is formulated within the framework of the poroplasticity of porous media. The porous medium is saturated by liquid water and a gas mixture. Two yield surfaces are used to describe plastic deformations respectively related to stress and suction variations. Based on some macroscopic assumptions, a modified Bishop type effective stress tensor is used for the modelling of plastic deformations due to stress loading. The model is then extended to include damage modelling during shrinkage. Damage evolution law is proposed as a function of tensile strains. Numerical simulations by using the model are presented and compared with experimental results.

1. Introduction

The analysis of concrete structures durability is based on the investigation of the material long term behaviour. Such a behaviour is influenced by mechanical, hydric and thermal actions applied to structures. The coupling between these actions makes more difficult the elaboration of a reliable model. In spite of many studies available so far on the estimation of deformation due to shrinkage of concrete, it exists very few results concerning the impact of drying shrinkage on the mechanical behaviour of concrete. Further, the durability of structures is also conditioned by the material responses to shocks and impacts. Indeed, the degradation due to microcracks induced by high me-
Mechanical loading can reduce life time of certain structures: nuclear power plant, nuclear waste storage facilities. Furthermore, shrinkage induces differed strains, which are able to significantly reduce the tension in prestress cables. In order to understand the behaviour of concrete and then propose a predictive modelling, it is necessary to investigate the influence of drying shrinkage on failure behaviour of concrete. Indeed, such an influence is usually not taken into account so far in many computer codes, probably due to the bad knowledge of degradation processes of concrete under hydric stresses [1].

This paper is composed of two main parts. In the first part, some experimental results are presented to show the importance of taking into account the coupling “damage – drying shrinkage” for a reliable concrete structures modelling. Indeed, the deformation induced by disturbance of hydric equilibrium between the inside of material body and the exterior surface are partially prevented both by granulates and differential shrinkage between different points of structure. This phenomenon causes microcracking of concrete by exceeding tensile strength of material. This microcracking will have an strong influence on the damage process of material and then its failure behaviour [2,3]. We present first the experimental investigation on drying shrinkage and various features taken into account. Then, uniaxial compression tests performed on samples at different ages are analysed in terms of material elastic stiffness degradation.

The second part is devoted to constitutive modelling. As a first stage work, a simple elastoplastic model coupled with isotropic damage is proposed for unsaturated cohesive geomaterials (concrete and rocks). During the last thirty years, many advances have been performed for unsaturated soils and clays both in experimental investigation and constitutive modelling, for instance [4, 5, 6, 7, 8, 9]. However, significant efforts are still necessary for constitutive modelling of unsaturated hard rocks and concrete. This is probably related to high difficulties in experimental testing. The common point of these materials is the possible microcracking due to change of hydric conditions and the coupling between plastic deformation and induced damage. On the other hand, some theoretical developments have been achieved on the mechanics of saturated and unsaturated porous materials. Macroscopic theories and micromechanical approaches have been proposed in order to relate microscopic structures to macroscopic responses [10, 11, 12, 13]. In particular, a general poroplasticsity theory of saturated porous media is formulated by [10] in the framework of thermodynamics of open systems. This theory is extended by [11] to unsaturated soils. On the basis of these theories, an isotropic elastoplastic damage model is proposed for unsaturated cohesive geomaterials (concrete and rocks). The model is formulated in the framework of poroelasticity and continuum damage mechanics. Two yield surfaces are used for the description of plastic deformation respectively due to stress and suction variations. The damage of material is described by the model proposed by Mazars [14]. The proposed model is implemented into a FEM code devoted for the numerical solution of hydro-mechanical problems in saturated and unsaturated porous media. Simulations of uniaxial compression tests with different water saturation degrees are provided. An example of modelling of drying shrinkage process in a cylinder concrete sample is finally presented.
2. Experimental investigation on drying shrinkage

2.1. Definition and principles
The total shrinkage of concrete can be decomposed of several parts \([2, 15, 16]\) and the influence of each part is correlated with the concrete composition \([17, 18, 19, 20]\): endogenous shrinkage, thermal shrinkage and desiccation shrinkage. The thermal shrinkage is caused by cooling process after the formation of concrete whose temperature increases by effect of cement hydration. Self-desiccation shrinkage is from the internal material drying by consuming water in pores during hydration process. The endogenous shrinkage is defined as the sum of Le Châtelier contraction and self-desiccation shrinkage \([21, 22, 23, 24]\). Finally, the desiccation shrinkage is induced by the disturbance of hydric equilibrium between the inside body of concrete and its exterior surface. This hydric gradient causes the leak-off of free water from concrete. Such a change of water content induces a compressive deformation of skeleton material due to increase of capillary pressure. However, this deformation is partially prevented by both gravels and heterogeneous hygrometric states in concrete body. A tensile stress appears in cement matrix. There is initiation and propagation of microcracks when the tensile stress exceeds the material strength. In this work, we present a preliminary study of the influences of such microcracks on the macroscopic elastoplastic and damage behaviour of concrete. In order to capture this desiccation process and its consequences, it is necessary to design a specific laboratory testing procedure.

2.2. Concrete choice
The choice of concrete material is made in a manner to maximise shrinkage deformation in a relative short time. There is a relationship between drying shrinkage and the ratio W/C (water/cement) \([19, 25]\). We have chosen a concrete with a high value of the ratio W/C. Indeed, such a material presents a high permeability facilitating leak-off of water from sample. In addition, a strong shrinkage should be observed in concrete with a small diameter of biggest gravel and a high ratio water/cement. Consequently, we have used a concrete with 8 mm diameter of biggest gravel and a W/C equal to 0.6. The cement is CPJ/CEMIIb 32.5. The composition of concrete is shown in Table 1. This composition is used in order to obtain an uniaxial compression strength less than 30 MPa at 28 days.

<p>| | |</p>
<table>
<thead>
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<td>Cement</td>
<td>324 kg/m³</td>
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<td>Water</td>
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<tr>
<td>Gravel 4/8 mm</td>
<td>1110 kg/m³</td>
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<td>Sand 0/4 mm</td>
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<tr>
<td>W/C</td>
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Table 1: Concrete composition
With this ratio W/C=0.6, the cement matrix structure is formed of big diameter pores [3, 18, 19] and then allows a high drying kinetics. After 28 days, the cement porous structure (W/C = 0.6) is completed [16], and this lets us to consider that hydric exchange will occur with a quasi constant porosity for each specimen even if the failure strength continues to increase in time (see Fig. 1). This figure presents the variation of uniaxial compression strength versus time. The referential origin of time corresponds to the end of concrete cure (see paragraph 2.4). Consequently, the increase of mechanical strength is both due to the suction induced by leak-off of water from pores, and maturation process due to hydration of cement. We can consider that suction effect is bigger than maturation effect because, for W/C = 0.6, porous structure of cement does not change after 28 days [16].

![Figure 1: Evolution of concrete strength versus time](image-url)
These results (Fig. 1) are quite different from those obtained by [26, 27] in term of compressive strength evolution, but cure conditions are also very different. In Pihlajavaara tests [27], the mortar samples have been conserved during 3 years in order to obtain constant moisture in material. In the presented tests, moisture content of concrete is not homogeneous. The aim of the experimental investigation is to show effects of this heterogeneous moisture content on the evolutions of elastic parameters and the failure stress in concrete during drying. We can notice that Pihlajavaara has obtained an increase of flexural strength of mortar with the decrease of concrete moisture content. According to Pihlajavaara, a simple explanation of this strengthening effect is a development of capillary forces in porous structure of the mortar, with the decrease of relative humidity [27].

2.3. Samples preparation
The measuring of shrinkage should be made on small size samples. At the same time, the size of specimens for uniaxial compression tests should be large enough to keep the sense of representative volume element. We have used cylinder samples of 110 mm diameter and 220 mm height. For shrinkage measurement, the size of prismatic specimens is 40x40x160 mm$^3$.

2.4. Maturation process
The self-desiccation shrinkage occurs mainly during hydration [2, 23] and then can be neglected after 28 days. This has been confirmed in [19] where it has been observed that endogenous shrinkage stopped after 28 days while the drying shrinkage continued to progress. Colina and Acker have shown that the temperature is stabilised in a one meter size cube after 31 days [28]. It is then reasonable to assume that the temperature will become constant in our specimens after 28 days. Consequently, the thermal shrinkage can also be neglected.

We have conserved the specimens during 28 days in a pool at a 20°C constant temperature. in order to avoid the dissolution of portlandite in water, the cylinder specimens were kept inside the paper covers and the exchange of ions is then prevented. The prismatic specimens were conserved inside plastic bags emerged in water. After 28 days of curing, the specimens are taken off from water and conserved under a relative humidity of 60% ± 5% and a temperature of 21°C ± 1°C.

Furthermore, it exists an asymptotic stabilisation of desiccation shrinkage after 90 days [19, 28]. This lets us to consider that the main shrinkage is produced during the first three months of drying which is a reasonable duration for proposed study. The scale effect should also be taken into account in the shrinkage phenomenon of concrete [17, 24, 28]. Indeed, the internal variation of hydric states are dependent on sample size. Bigger is the structure size slower are the hydric state changes. Inversely, there is a rapid variation of water content in structures with small characteristic size. Accordingly, the effect of drying shrinkage on the mechanical behaviour of concrete may be detectable only in
small size structures [2, 28]. This justifies that the used specimen size is judicious with respect to that we want to show.

![Figure 2: Evolution of shrinkage versus the loss in mass for 3 prismatic specimens](image)

In order to investigate the influence of shrinkage on mechanical behaviour, we assume that thermal, hydric and mechanical effects on microcraking are not coupled [28]. Uniaxial compression tests are carried out during the time to study the influence of drying on the elastic properties degradation. At the same time, the weight of specimen are regularly measured. We can summarise the experimental investigation program as follows:

- Measuring of shrinkage on specimens of 40x40x160 mm³, during 70 days
- Measuring of weight on specimen 40x40x160 mm³, during 70 days
- Measuring of weight on specimen $\Phi 110 \times h 220$ mm, during 70 days
- Uniaxial compression tests with unloading cycles on cylinder specimens $\Phi 110 \times h 220$ mm, during 70 days.
All the specimens were prepared the same day in two times due to the capacity of machine.

2.5. Shrinkage results
To take into account the cure procedure, the following hypothesis is assumed: when samples are taken out water, they are saturated. The global water content is then deduced from the loss in mass of specimens.

Figure 3: Evolution of the relative mass during time for 11*22 cylindrical specimens tested at different time
Figure 2 shows the evolution of shrinkage strain in three specimens as a function of loss in mass during 70 days. The results are similar to those obtained by Granger [29]. Two phases can be distinguished. The first part of the curves corresponds to the rapid drying of specimen surface. This is accompanied with superficial microcracking which delays the progress of global shrinkage. In the second phase, there is proportionality between loss in mass and shrinkage strain, this evidences the effect of drying on concrete. If the measurement was continued after 70 days, a third phase would be obtained where the shrinkage strain would tend to stabilise while the loss in mass continues to evolve. Such a phase is not interesting here because in general, no further growth of microcracks is observed during this period.

Figure 3 gives the normalised masses of 5 cylinder specimens as functions of time. The normalised mass is defined as the ratio between the current mass with respect to the initial mass of specimen. These evolutions correspond to classic results reported in literature. However, it is interesting to notice that these specimens have been tested under mechanical loading until failure state at different dates (7, 21, 41, 56 and 69 days after exit from water). After each test, the specimen is carefully conserved in order to pursue the measurement of loss in mass. The influence of mechanical loading on the kinetics of drying is relatively small. It appears that the shrinkage process is rather influenced by diffused microcracks than by macroscopic cracks formed under mechanical loading. Accordingly, it seems to be reasonable to develop a constitutive modelling of diffused damage using an isotropic or anisotropic damage variable will be necessary. The evolution of damage should depend on mechanical loading and variation of hygrometry conditions.

2.6. Uniaxial compression tests during time
The uniaxial compression tests have been carried out with the help of a hydraulic machine (Instron with 500 kN of capacity), in displacement controlled with a strain rate of $2 \times 10^{-4}$ s$^{-1}$. The axial applied force and overall axial deformation are monitored with a LVDT. The axial stress and strain are then deduced and presented on Figure 4. Unloading–reloading cycles are also shown in this figure. During drying, the water content is no more uniform in the specimen, this leads to the definition of an overall elastic stiffness of specimen rather than the elastic modulus of concrete material. The elastic stiffness is strongly influenced by growth of microcracks during drying shrinkage.

In order to compare the results obtained from different specimens, we propose to define three dimensionless quantities, the initial normalised stiffness, normalised strain and damage. The initial normalised stiffness corresponds to the stiffness measured from the third loading-unloading cycle in each test (the corresponding axial stress is 9 MPa), divided by the maximal value of this stiffness obtained in all specimens. In the present work, such a maximal value is obtained from the specimen dried during 7 days. The normalised strain is calculated from the residual strain after each loading cycle, divided
by the strain obtained at the peak stress in each test (see figure 4). Finally, the damage is defined as the ratio between the current elastic stiffness and the initial stiffness.

Figure 4: Stress – strain behaviour. Definitions of mechanical parameters
Figure 5: Evolution of dimensionless stiffness versus the loss in mass

Figure 5 shows the evolutions of dimensionless initial stiffness in cylinder specimens as functions of loss in mass. Each point represents the average value of two tests performed the same day. It is clear that the drying process induces a diminution of elastic stiffness of concrete. This fact should be taken into account in numerical modelling.

The evolutions of damage are presented on Figure 6 as a function of dimensionless strain for 4 cylinder specimens tested respectively at 2, 24, 44 et 69 days. It seems obvious that there is a stronger kinetics of damage in specimens dried for a longer time. Therefore, it appears necessary to account for the coupling between drying shrinkage and mechanical behaviour, in particular induced damage and plastic deformation of concrete.
3. Concrete modelling during drying

3.1. General considerations
The porous concrete studied is assumed to be isotropic in this paper. The model is formulated in the framework of thermodynamics of open systems. The porous medium is assumed to be saturated by a liquid phase (noted by index $w$) and a gas mixture (or air) phase (noted by index $g$). The deformation of the porous medium is identified by that
of the skeleton and there are fluid mass exchanges with the exterior due to external actions. The state variables to be used are the strain tensor of skeleton $\mathbf{\varepsilon}_{ij}$, the volumetric change of water content $(\phi_w - \phi_{w0})$ and gas content $(\phi_g - \phi_{g0})$, the damage variable $d$, the internal state variables for plastic hardening $\chi_k$, and the temperature $T$. The hypothesis of small transformations is used throughout the paper. The following decompositions are assumed:

$$\mathbf{\varepsilon}_{ij} = \mathbf{\varepsilon}_{ij}^e + \mathbf{\varepsilon}_{ij}^p$$

(1)

$$\phi_w - \phi_{w0} = \phi_w^e + \phi_w^p$$

(2)

$$\phi_g - \phi_{g0} = \phi_g^e + \phi_g^p$$

(3)

In this paper, the gas is assumed to behave as an ideal gas and the liquid water as a standard compressible fluid. The application of thermodynamics laws to this open system leads to the following inequality [10]:

$$\sigma_{ij} d\varepsilon_{ij} + p_g d\phi_g + p_w d\phi_w - S_s dT - d\Psi_s \geq 0$$

(4)

where $S_s$ and $\Psi_s$ are the skeleton entropy and free energy per unit initial volume of the bulk material. The standard differentiation of the free energy function in (4) yield in the following state equations

$$\sigma_{ij} = \frac{\partial \Psi_s}{\partial \varepsilon_{ij}}, \quad p_g = \frac{\partial \Psi_s}{\partial \phi_g^e}, \quad p_w = \frac{\partial \Psi_s}{\partial \phi_w^e}, \quad S_s = -\frac{\partial \Psi_s}{\partial T}$$

(5)

The irreversible processes involving plastic flow and damage evolution have to satisfy the fundamental inequality as follows:

$$\sigma_{ij} d\varepsilon_{ij}^p + p_g d\phi_g^p + p_w d\phi_w^p - \frac{\partial \Psi_s}{\partial \chi_k} d\chi_k - \frac{\partial \Psi_s}{\partial d} d(d) \geq 0$$

(6)

The elastic damage behaviour of partially saturated concrete is obtained by extending the non linear elastic model proposed by [10] and the isotropic damage model by [14]. The constitutive equations can be expressed in the following incremental form:

$$d\sigma_{ij} = C_{ijkl}^b d\varepsilon_{kl} - \beta (S_w d\phi_w + (1 - S_w) d\phi_g)$$

(7)

$$C_{ijkl}^b = (1 - d)C_{ijkl}^{b0} - (C_{ijkl}^{b0} : \mathbf{\varepsilon}^e) \otimes \frac{\partial \mathbf{d}}{\partial \mathbf{\varepsilon}^e}$$

(8)
where \(C^{hT}(d)\) is tangent elastic stiffness tensor of damaged material, \(b\) is the Biot’s coefficient of porous medium and \(S_w\) the water saturation degree. The micromechanical interpretation of this incremental form has been discussed by [13] by using a homogenization approach. This is exact when the surface traction is equal to the capillary pressure at the gas-solid interface.

The complementary laws for plastic deformation are composed of the yield function, plastic potential and hardening law. The exact forms of these functions should be identified from relevant experimental data. The yield function and plastic potential should be expressed in the space of the conjugate internal variables \((\sigma_{ij}, p_G, p_w, Y_k)\). In the case of unsaturated materials, experimental results have shown that plastic deformation can develop due to the variation of suction under constant stresses. Therefore, two distinct plastic mechanisms are usually identified, respectively called stress related and suction related plastic deformations. Furthermore, the stress related plastic deformation is strongly influenced by the water saturation degree (or the variation of capillary pressure). Therefore, two yield functions can be formally expressed:

\[
\begin{align*}
    f_\sigma(\sigma_{ij}, Y_k, p_G, p_w) &= 0 \\
    f_s(p_c, Y_k; S_w) &= 0
\end{align*}
\]

where \(p_c\) is the capillary pressure (suction) defined by:

\[
p_c = p_G - p_w
\]

Similarly the plastic potentials corresponding to the two yields functions may be as follows:

\[
\begin{align*}
    g_\sigma(\sigma_{ij}, Y_k, p_G, p_w) &= 0 \\
    g_s(p_c, Y_k; S_w) &= 0
\end{align*}
\]

The plastic flow rules for each mechanism are given by \((\alpha = \sigma, s)\):

\[
\begin{align*}
    d\varepsilon^p_{ij} &= \Delta\lambda_\alpha \frac{\partial g_\alpha(\sigma_{ij}, p_G, p_w, Y_k; S_w)}{\partial \sigma_{ij}} \\
    d\phi^p_w &= -\Delta\lambda_\alpha \frac{\partial g_\alpha(\sigma_{ij}, p_G, p_w, Y_k; S_w)}{\partial p_w}
\end{align*}
\]
\[ d\phi^\alpha_g = -\Delta \alpha \frac{\partial g_{\alpha}}{\partial p_{\alpha}}(\sigma_{ij}, p_g, p_w, Y_k, S_w) \]  

(17)

The total plastic strains are calculated by:

\[ de^p_{ij} = de^p_{ij} + de^{\sigma p} \]

(18)

In practice, the experimental determination of the yield function and plastic potential for stress-related mechanism is not easy. The influence of water saturation degree on stress related plastic deformation is taken into account by introducing suitable empirical relations to describe the variation of some material parameters (the cohesion in most case) as a function of suction [7]. In order to develop a more consistent approach, we propose to introduce an extension of the concept of plastic effective stress for saturated media [10] to partially saturated materials. Namely, the Bishop effective stress used in the elastic domain is generalised to the plastic domain:

\[ \sigma^p_{ij} = \sigma_{ij} + \beta(S_w p_w + (1 - S_w) p_g) \delta_{ij} \]  

(19)

The parameter \( \beta \) should be identified from relevant experimental data. Therefore, the yield function \( f_\sigma \) and plastic potential \( g_\sigma \) for the stress related plastic deformation are formulated as functions of this ‘plastic’ effective stress in order to involve influence of capillary pressure.

3.2. Presentation of a particular model

The yield function for stress related plastic deformation is based on the classic Drucker-Prager criterion:

\[ f_\sigma = q\left(\cos \theta - \frac{t_c}{\sqrt{3}} \sin \theta\right) + \alpha_p \left(p^p - C^p_{sat}\right) = 0 \]  

(20)

where \( p^p \) is the plastic effective mean stress defined as:

\[ p^p = \frac{\sigma^p_{kk}}{3} = \frac{\sigma_{kk}}{3} + \beta[S_w p_w + (1 - S_w) p_g] = \frac{\sigma_{kk}}{3} + \beta[p_g - S_w p_c] \]  

(21)

and \( q \) is the deviatoric stress and \( \theta \) the Lode’s angle.

\[ q = \sqrt{\frac{3}{2}} S_{ij} S_{ij} \quad \text{with} \quad S_{ij} = \sigma_{ij} - \frac{\sigma_{kk}}{3} \delta_{ij} \]  

(22)
\[
\theta = \frac{1}{3} \text{Arc sin} \left[ \frac{3\sqrt{3} \cdot J_3}{2 \cdot \sqrt{J_3^2}} \right], \quad J_2 = \frac{1}{2} S_{ij} S_{ij}, \quad J_3 = \text{det} \mathbf{S}
\] (23)

The parameter \( C_{sat}^p \) is the material cohesion for saturated condition, and \( t_c \) is used to describe the strength asymmetry in compression and extension. The function \( \alpha_p \) defines mobilised friction used as plastic hardening function. In order to account for coupling between the stress and suction related plastic mechanisms, we choose the total equivalent plastic strain \( \bar{\varepsilon}_p \) as plastic yield variable:

\[
\alpha_p = \alpha^p_0 - \left( \alpha^p_p - \alpha^p_0 \right) e^{-b \bar{\varepsilon}_p}, \quad \bar{\varepsilon}_p = \sqrt{\varepsilon^p_{ij} \varepsilon^p_{ij}}
\] (24)

The parameters \( \alpha^p_0 \) and \( \alpha^p_p \) are respectively the initial and ultimate value of the hardening function and \( b \) controls the kinetics of plastic hardening.

Based on experimental results obtained from most geomaterials, a non-associative plastic flow is chosen by using the following plastic potential:

\[
g_{\sigma} = q + \beta_p \sigma^p
\] (25)

with \( \beta_p = \beta^p_0 + 2b_1 \gamma_p \) and \( \beta_p \leq \beta^p_{\text{max}} \) (26)

The parameters \( \beta^p_0 \) and \( \beta^p_{\text{max}} \) are respectively the initial and asymptotic value of \( \beta_p \) and \( b_1 \) controls the evolution of material plastic dilatancy.

### 3.3. Suction-related plasticity

The suction related plastic flow is assumed to be spherical and able to describe the material swelling and compaction due suction change under constant stresses. Further, an associated flow rule is adopted. The yield function is defined by:

\[
f_s = p_c - s^p = 0 \quad \text{and} \quad g_s = f_s
\] (27)

The suction plastic hardening is a function of the total plastic volumetric strain \( \varepsilon^p_v \):

\[
s^p = s^p_0 \ e^{c \varepsilon^p_v}
\] (28)

\( s^p_0 \) define the initial value of yielding suction and \( c \) controls suction hardening kinetics.
The total plastic strain increment $d\varepsilon_{ij}^p$ is then calculated from the contributions of two mechanisms:

$$d\varepsilon_{ij}^p = d\varepsilon_{ij}^{p_{\sigma}} + \frac{1}{3} d\varepsilon_{ij}^{p_s} \delta_{ij} = \Delta \lambda_{\sigma} \frac{\partial \sigma_{ij}}{\partial \lambda_{ij}} - \Delta \lambda_{s} \frac{1}{3} \frac{\partial \lambda_{ij}}{\partial \sigma_{ij}} \delta_{ij}$$

Two plastic multipliers $\Delta \lambda_{\sigma}$ and $\Delta \lambda_{s}$ are obtained through plastic consistency conditions. The figure 7 shows a view of the yield surfaces in the $(p, q, p_c)$ space.

![Stress related yield surface in the saturated state](image)

**Figure 7:** A view of the mechanical and capillary yield surfaces in the $(p, q, p_c)$ stresses space.

### 3.4. Damage modelling

In this paper, only an isotropic damage is considered for the reason of simplicity. The modelling is based on the damage model proposed by Mazars for concrete materials [14]. In order to take into account the contribution of plastic deformation to damage evolution, it is assumed that the damage is controlled by the total equivalent tensile strain:
As in the model of Mazars, the total damage is composed of components caused respectively by tensile $d_t$ and compressive stress $d_c$:

\[ d = \alpha_t d_t + (1 - \alpha_t) d_c \]  

(31)

The parameter $\alpha_t$ defines the ration between damages by tensile and compressive stresses and calculated by the following relation:

\[ \alpha_t = \sum_i \frac{H(\varepsilon_t^i) \varepsilon_t^i (\varepsilon_t^i + \varepsilon_c^i)}{(\varepsilon^*)^2} \]  

(32)

where $H(x)$ is the Heaviside function of principal strain. $\varepsilon_t^*$ and $\varepsilon_c^*$ are respectively elastic strains due to tensile (positive) and compressive (negative) principal stress $\sigma_i$ under a given damage state. The damage evolutions due to tensile and compressive loading are described by the following relations proposed by Mazars [14]:

\[ d_t = 1 - \frac{\varepsilon_{d0}(1 - A_t)}{\varepsilon_M} - \frac{A_t}{\exp[B_t(\varepsilon_M - \varepsilon_{d0})]} \]  

(33)

\[ d_c = 1 - \frac{\varepsilon_{d0}(1 - A_c)}{\varepsilon_M} - \frac{A_c}{\exp[B_c(\varepsilon_M - \varepsilon_{d0})]} \]  

(34)

where $\varepsilon_{d0}$ is the initial damage limit and $\varepsilon_M$ the maximal total equivalent tensile strain. Four parameters $A_t$, $B_t$, $A_c$, and $B_c$ control damage evolution kinetics under tensile and compressive loading.

The table 2 gives the representative set of parameters for a concrete.
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<tr>
<th>Parameter</th>
<th>Value</th>
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Table 2: Model parameters used in the simulations.

4. Numerical Simulations

The proposed model has been introduced in the finite element code MPPSAT which is devoted for resolution of coupling problems in partially saturated porous media. The model’s parameters are determined from the previously performed tests on saturated samples. The parameters are given in the table 2. The lack of hydric experimental data forces us to take the hydric parameters from [30]. Indeed, some drying-wetting tests are needed to determine $s_0^p$ and $c$. 
Figure 8: Numerical simulations of an uniaxial compression test.

Figure 8 shows the simulation of an uniaxial compression test by using the model for a sample with a water saturation degree of 85 %. The basic elastoplastic damage behaviour of the concrete is correctly reproduced.
The influence of water saturation degree on the mechanical responses of concrete is clearly shown in Figure 9 by the proposed model. The mechanical strength increases and the material behaviour becomes more brittle in nature when the water saturation degree decreases.
gree decreases due to the effect of capillary force. However, as mentioned above, the parameter for hydric coupling are not determined from relevant experimental data, the increase in mechanical strength predicted by the model appears too much with respect to available results. These results are presented only to show the capacity of the model. Simulations with realistic parameters will be performed later after complementary laboratory tests. It is important to notice that the main effects due to desiccation are correctly described by the model.

In order to illustrate the drying evolution of a structure element, a simple example is investigated and shown on Figure 10. A cylinder body is analysed under axi-symmetric conditions. The boundary conditions are as follows: three faces are maintained to a relative humidity of 100 % while a relative humidity of 60 % is prescribed on the last face.

![Figure 10: Boundary conditions for drying process simulation of concrete.](image)

The results obtained are presented on Figure 11. The evolution of internal relative humidity at different times (1h, 10 h et 50 h) are reported for different radius of sample. The model seems to be able to reproduce the drying process of concrete. The hydric state in the sample becomes heterogeneous during time, this leads to a differential desiccation shrinkage and possibility of induced damage.
Figure 11: Humidity evolutions in the drying simulation.
5. Conclusions

An original experimental investigation is proposed in this paper in order to study the influence of shrinkage on the mechanical behaviour of concrete. Uniaxial compression tests have been conducted on samples submitted to different periods of drying. The obtained results have clearly shown a coupling between shrinkage and damage. A simple isotropic elastoplastic damage model is proposed for partially saturated porous geomaterials. This model is able to describe fundamental mechanical behaviours of concrete due to mechanical and hydric stresses. A simple example of application is presented to show the applicability of such a model for durability analysis of concrete structure. More laboratory tests are still necessary for a better understanding of phenomena and the validation of the constitutive model.

6. REFERENCES
