Abstract
The overall objective of this paper is to evaluate four different procedures for predicting the concrete breakout capacity of tensile anchors under static and dynamic loading, and in uncracked and cracked concrete. An existing public-domain data base of tensile anchors was evaluated and updated. Observed capacities of tensile anchors failing by concrete breakout were compared with the predictions of four methods: the 45-Degree Cone Method; the CC Method, and a variation on it; and a “Theoretical Method.” Each predictive method was then evaluated using Monte Carlo analyses to predict the probability of failure by concrete breakout, using the design framework of ACI 349-90, Appendix B “Steel Embedments.” [1]

1. Introduction
The objective of this research was to provide the US Nuclear Regulatory Commission (NRC) with a comprehensive document that could be used to establish regulatory positions regarding fastening to concrete. Tensile behavior of anchors under static and dynamic loading in uncracked and cracked concrete, and for cast-in-place, undercut, sleeve and expansion anchors, is evaluated using the design framework of ACI 349-90, and four possible predictive equations for concrete breakout:

1) the 45-Degree Cone Method;
2) the Concrete Capacity Method (CC Method), and a variation on that method; and
3) a “Theoretical Method” related to the CC Method.

Available test data are evaluated and organized by failure mode, using descriptions and photographs presented by the original researchers. Each set of design provisions is evaluated based on the following criteria [2]:

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**BEHAVIOR OF TENSILE ANCHORS IN CONCRETE: STATISTICAL ANALYSIS AND DESIGN RECOMMENDATIONS**

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1) An ideal design method should give ratios of observed to predicted capacity showing no systematic error (that is, no variation in ratios with changes in embedment depth), high precision (that is, little scatter of data).

2) An ideal design method should have acceptably low probabilities of failure in the overall design framework in which it is to be used.

2. Test Data for Tensile Anchors in Concrete

The public-domain data base used for this purpose is maintained by ACI Committees 349 and 355, and comes from many contributors, including Dr. Werner Fuchs (University of Stuttgart), Drilco Industries, Inc., Prof. Peter Carrato (Bucknell University), The University of Texas at Austin, Hilti AG, and various members of ACI Committees 349 and 355. The data base contains data for tensile breakout failures only.

3. Background

General information on anchor types and behavior is given in CEB (1991) [3]. Essential information is summarized here.

**Tensile Breakout Capacity by 45-Degree Cone Method**

The 45-Degree Cone Method assumes that a constant tensile stress of \(4\sqrt{f_c'}\) acts on the projected area of a 45-degree cone radiating towards the free surface from the bearing edge of the anchor (Figure 1).

![Figure 1](image-url)  
*Tensile breakout cone as idealized by 45-degree Cone Method*
For a single tensile anchor far from edges, the cone breakout capacity is determined by:

\[
T_o = 4 \sqrt{f'_c \pi h_{ef}^2 \left(1 + d_h / h_{ef}\right)} \quad \text{lb} \quad (1a)
\]

\[
T_o = 0.96 \sqrt{f'_c \pi h_{ef}^2 \left(1 + d_h / h_{ef}\right)} \quad \text{N} \quad (1b)
\]

where:
\(f'_c\) = specified concrete compressive cylinder strength (psi in US units, MPa in SI units);
\(d_h\) = diameter of anchor head (inch in US units, mm in SI units); and
\(h_{ef}\) = effective embedment (inch in US units, mm in SI units).

If the cone is affected by edges \((c < h_{ef})\) or by an adjacent concrete breakout cone, the breakout capacity is:

\[
T_n = A_N A_o \quad (2)
\]

where:
\(A_N\) = actual projected area of failure cone or cones;
\(A_{No}\) = projected area of a single cone unaffected by edges
\(= \pi h_{ef}^2 \left(1 + d_h / h_{ef}\right)\).

**Tensile Breakout Capacity by Concrete Capacity Method (CC Method)**

The CC Method [4] computes the concrete breakout capacity of a single tensile anchor far from edges as:

\[
T_o = k \sqrt{f'_c h_{ef}^{1.5}} \quad (3)
\]

where:
\(T_o\) = tensile breakout capacity;
\(k\) = constant; for anchors in uncracked concrete the mean values originally proposed based on previous tests are: 35 for expansion and sleeve anchors, 39 for undercut and headed anchors, in US units; or 13.48 for expansion and sleeve anchors, 15 for undercut and headed anchors, in SI units;
\(f'_c\) = specified concrete compressive strength \((6 \times 12\) cylinder\) (inch in US units, MPa in SI units.);
\(h_{ef}\) = effective embedment depth (inch in US unit, MPa in SI unit).

In the CC Method, the breakout body is idealized as a pyramid with an inclination of about 35 degrees between the failure surface and the concrete member surface (Figure 2).
As a result, the base of the pyramid measures $3h_{ef}$ by $3h_{ef}$. If the failure pyramid is affected by edges or by other concrete pyramids, the concrete capacity is calculated according the following equation:

$$T_n = \frac{A_N}{A_{No}} \psi_2 T_{no}$$  

(4)

where:

- $A_{No}$ = projected area of a single anchor at the concrete surface without edge influences or adjacent-anchor effects, idealizing the failure cone as a pyramid with a base length of $s_{cr} = 3h_{ef}$ ($A_{no} = 9h_{ef}^2$) (See Appendix A of Reference 2);
- $A_N$ = actual projected area at the concrete surface;
- $\psi_2$ = tuning factor to consider disturbance of the radially symmetric stress distribution caused by an edge,
  - $= 1$, if $c_1 \geq 1.5h_{ef}$;
  - $= 0.7 + 0.3 \frac{c_1}{1.5h_{ef}}$, if $c_1 \leq 1.5h_{ef}$;

where:
- $c_1$ = edge distance to the nearest edge.

**Tensile Breakout Capacity by “Theoretical Method”**

The “Theoretical Method” is based on linear elastic fracture mechanics, including the size effect [5]. Tensile breakout capacity is:
\[ N_n = \frac{k \cdot \sqrt{f_{cc} \cdot h_{ef}^2}}{\left(1 + \frac{h_{ef}}{50}\right)^{0.5}} \]  

(5)

where:
- \( N_n \) = predicted concrete tensile breakout capacity (kN)
- \( f_{cc} \) = actual tested strength of a 200-mm concrete cube (MPa)
- \( h_{ef} \) = effective embedment (mm)
- \( k \) = 2.75 for undercut and cast-in-place anchors, and 2.5 for expansion and sleeve anchors.

**Tensile Breakout Capacity by the Variation on the CC Method**

As a result of previous work in ACI Committees 318 and 349 (Subcommittee 3), it has been proposed to modify the CC Method slightly, changing the exponent of the embedment depth from 1.5 to 1.67 for effective embedments of 250 mm (9.84 in) or greater, and changing the leading coefficient appropriately.

**Effects of Dynamic Tensile Loading and Cracks on Tensile Breakout Capacity**

In this research, predicted tensile breakout capacities under static loading were multiplied by a dynamic factor equal to 1.25 for undercut, cast-in-place, and sleeve anchors, and equal to 1.0 for expansion anchors [6, 7, 8]. Capacities in uncracked concrete were multiplied by a crack factor equal to 0.9 for undercut and cast-in-place anchors, equal to 0.7 for sleeve and expansion anchors [5, 7, 8, 9].

**4. Statistical Evaluation of Database (static, uncracked)**

The database for static testing on anchors in uncracked concrete comprises 1566 tests:

- a) Single tensile anchors, effective embedment \( \leq 188 \) mm, no edge effects (1130 tests);
- b) Single tensile anchors, effective embedment \( > 188 \) mm, no edge effects (77 tests);
- c) Single tensile anchors, effective embedment \( \leq 188 \) mm, edge effects (137 tests);
- d) Single tensile anchors, effective embedment \( > 188 \) mm, edge effects (33 tests);
- e) Tensile 2- and 4-anchor groups, effective embedment \( \leq 188 \) mm, no edge effects (170 tests); and
- f) Tensile 4-anchor groups, effective embedment \( > 188 \) mm, no edge effects (19 tests).

Means and coefficients of variation for ratios of observed to predicted capacity are shown in Table 1. All are for static loading in uncracked concrete.
Examination of Table 1 shows that the CC Method and the Theoretical Method usually are more accurate than the 45-Degree Cone Method (mean values closer to unity), and have less scatter (smaller COV). Since the 45-Degree Cone Method generally gave a higher COV than both the CC and Theoretical Methods, it was decided to exclude it for analysis of other cases.

5. Probabilities of Failure associated with each Breakout Formula

Using the overall ratios of concrete breakout capacity, appropriately approximated by normal distributions (Appendix B of Reference 2), probabilities of failure were computed for an assumed statistical distribution of loads, and probabilities of brittle failure were computed independent of load, for single anchors designed according to each method for predicting concrete breakout capacity. This statistical evaluation was carried out using the Monte Carlo approach, and assuming the ductile design framework and the load and understrength factors of ACI 349-90, Appendix B [10, 11].

Probabilities of Failure under Known Loads, Static Loading, Uncracked Concrete

Results of the statistical analyses are summarized in Table 2. Higher values of $\beta$ indicate lower probabilities of failure.
Because the probability of failure associated with the 45-Degree Cone Method was consistently higher than that of the CC Method or the Theoretical Method, it is not investigated further here.

Probabilities of Failure for Other Cases (Dynamic Loading, Cracked Concrete, or Both)
For known loads, probabilities of failure with the CC Method and the Theoretical Method are given in Table 3. All results are for single anchors, shallow embedment, no edge effect). These results are somewhat different from those of Farrow et al. [10, 11], because the anchors were categorized differently in that work (edge distance / embedment, spacing / embedment), and because this work used a more extensive data base.
Table 3  Probability of failure under known loads for different cases of tensile anchors, ductile design approach, Category One

<table>
<thead>
<tr>
<th>ANCHOR CASE</th>
<th>CC METHOD</th>
<th>THEORETICAL METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability of Failure</td>
<td>β</td>
</tr>
<tr>
<td>dynamic loading, uncracked concrete, cast-in-place and undercut</td>
<td>2.94E-13</td>
<td>7.20</td>
</tr>
<tr>
<td>dynamic loading, uncracked concrete, expansion and sleeve</td>
<td>9.78E-08</td>
<td>5.20</td>
</tr>
<tr>
<td>static loading, cracked concrete, cast-in-place and undercut</td>
<td>2.10E-08</td>
<td>5.48</td>
</tr>
<tr>
<td>static loading, cracked concrete, expansion and sleeve</td>
<td>1.60E-03</td>
<td>2.95</td>
</tr>
<tr>
<td>dynamic loading, cracked concrete, caste-in-place and undercut</td>
<td>1.58E-08</td>
<td>5.53</td>
</tr>
<tr>
<td>dynamic loading, cracked concrete, expansion and sleeve</td>
<td>3.62-06</td>
<td>4.49</td>
</tr>
</tbody>
</table>

Probabilities of Brittle Failure Independent of Load, Ductile Design Approach
Probabilities of brittle failure independent of load are given in Table 4.

Probabilities of Brittle Failure Independent of Load for Other Cases (Dynamic Loading, Cracked Concrete, or Both)
Probabilities of brittle failure independent of load are given in Table 5.
Table 4  Probabilities of brittle failure independent of load for different categories of tensile anchors, ductile design approach, Static, Uncracked

<table>
<thead>
<tr>
<th>ANCHOR CATEGORY</th>
<th>CC METHOD</th>
<th>45-DEG CONE METHOD</th>
<th>THEORETICAL METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability of Brittle Failure</td>
<td>$\beta$</td>
<td>Probability of Brittle Failure</td>
</tr>
<tr>
<td>single anchors, shallower embedments</td>
<td>0.178</td>
<td>0.922</td>
<td>0.066</td>
</tr>
<tr>
<td>single anchors, deeper embedments</td>
<td>0.088</td>
<td>1.36</td>
<td>0.369</td>
</tr>
<tr>
<td>single anchors, shallower embedments, edge effects</td>
<td>0.206</td>
<td>0.821</td>
<td>0.198</td>
</tr>
<tr>
<td>single anchors, deeper embedments, edge effects</td>
<td>0.0405</td>
<td>1.75</td>
<td>0.717</td>
</tr>
<tr>
<td>2- and 4-anchor groups, shallower anchors, no edge effects</td>
<td>0.107</td>
<td>1.24</td>
<td>0.125</td>
</tr>
<tr>
<td>4-anchor groups, deeper embedments, no edge effects</td>
<td>0.0621</td>
<td>1.54</td>
<td>0.273</td>
</tr>
</tbody>
</table>

Table 5  Probability of brittle failure independent of load for different cases of tensile anchors, ductile design approach, Category One

<table>
<thead>
<tr>
<th>ANCHOR CASE</th>
<th>CC METHOD</th>
<th>THEORETICAL METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability of Brittle Failure</td>
<td>$\beta$</td>
</tr>
<tr>
<td>dynamic loading, uncracked concrete, cast-in-place and undercut</td>
<td>7.16E-02</td>
<td>1.46</td>
</tr>
<tr>
<td>dynamic loading, uncracked concrete, expansion and sleeve</td>
<td>8.73E-02</td>
<td>1.36</td>
</tr>
<tr>
<td>static loading, cracked concrete, cast-in-place and undercut</td>
<td>1.19E-01</td>
<td>1.18</td>
</tr>
<tr>
<td>static loading, cracked concrete, expansion and sleeve</td>
<td>1.20E-01</td>
<td>1.17</td>
</tr>
<tr>
<td>dynamic loading, cracked concrete, caste-in-place and undercut</td>
<td>5.89E-02</td>
<td>1.56</td>
</tr>
<tr>
<td>dynamic loading, cracked concrete, expansion and sleeve</td>
<td>5.74E-02</td>
<td>1.61</td>
</tr>
</tbody>
</table>
These probabilities of failure are independent of the assumed statistical distribution of the loads. The ductile failure criterion, which requires actual steel fracture before concrete breakout, is quite severe, and these computed probabilities of brittle failure are conservative (high). Results from Tables 4 and 5 imply that the CC Method generally gives lower probabilities of brittle failure than the 45-Degree Cone Method and the Theoretical Method.

**Probabilities of Failure for the Variation on the CC Method, Known Loads**

Probabilities of failure under known loads are presented in Table 6. The probabilities of failure are of course identical in Anchor Categories One, Three and Five, since the method is identical to the CC Method for shallower embedments, so the table includes only deep-embedment categories. Probabilities of failure are clearly higher for the variation in the CC Method.

Table 6  Probability of failure under known loads for different categories of tensile anchors, ductile design approach, Static, Uncracked

<table>
<thead>
<tr>
<th>ANCHOR CATEGORY</th>
<th>CC METHOD</th>
<th>VARIATION ON CC METHOD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probability of Failure</td>
<td>β</td>
</tr>
<tr>
<td>single anchors, deeper embedments</td>
<td>3.27E-05</td>
<td>4.51</td>
</tr>
<tr>
<td>single anchors, deeper embedments, edge effects</td>
<td>1.70E-06</td>
<td>4.65</td>
</tr>
<tr>
<td>4-anchor groups, deeper embedments, no edge effects</td>
<td>5.36E-04</td>
<td>3.27</td>
</tr>
</tbody>
</table>

6. Conclusions

1) The CC Method and the Theoretical Method have a generally lower probability of failure under known loads, than the 45-Degree Cone Method. These results are consistent with those of Farrow et al. [10, 11]. The lower probability of failure is particularly striking for deeper embedments. The CC Method has a generally lower probability of failure under known loads, than the Theoretical Method.

2) The CC Method has a generally lower probability of brittle failure independent of load, than the 45-Degree Cone Method and the Theoretical Method.

3) The Variation on the CC Method, which uses an exponent of (5/3) for the effective embedment at deeper embedments, has higher systematic error and higher probabilities of failure than the CC Method. It has no technical justification.
7. Acknowledgement and Disclaimer

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8. References

1. ACI Committee 349, “Code Requirements for Nuclear Safety Related Concrete Structures,” American Concrete Institute, Detroit, 1990.


