SMEARED FRACTURE FE-ANALYSIS OF REINFORCED CONCRETE STRUCTURES – THEORY AND EXAMPLES

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Abstract
In the present paper the smeared fracture concept for the nonlinear finite element analysis of concrete and reinforced concrete structures is discussed. After a short introduction into the problems related to the smeared crack approach, a brief theoretical background of the finite element code MASA is presented. The code has been developed at the Institute of Construction Materials, University of Stuttgart, and is aimed to be used for the nonlinear smeared fracture finite element analysis of concrete and reinforced concrete structures. The used constitutive law (microplane model for concrete) and the so called localization limiters, which assure mesh objective results, are described. To demonstrate the ability of the code to realistically predict the ultimate capacity and failure mode of concrete and reinforced concrete structures, several examples are shown and briefly discussed.

1. Introduction
In recent years a significant progress in modeling of concrete like materials for general stress-strain histories has been achieved. Presently available models for concrete can roughly be classified in two categories: (i) Macroscopic models, in which the material behavior is considered to be an average response of a rather complex microstructural stress transfer mechanism and (ii) microscopic models, in which the micromechanics of deformations are described by stress-strain relations on the microlevel. No doubt, from the physical point of view microscopic models are more promising. However, they are computationally extremely demanding. Therefore, in practical applications macroscopic models have to be used.

At the macro scale the model has to correctly describe microstructural phenomena such as cohesion, friction, aggregate interlock and interaction of microcracks. Traditionally,
macroscopic models are formulated by total or incremental formulation between the $\sigma_{ij}$ and $\varepsilon_{ij}$ components of the stress-strain tensor, using the theory of tensorial invariants [1][2]. In the framework of the theory there are various possible approaches for modeling of concrete, such as theory of plasticity, plastic-fracturing theory, continuum damage mechanics, endocronic theory and their combinations of various form. Due to the complexity of concrete these models can not realistically represent the behavior of concrete for general three-dimensional stress-strain histories. Therefore, to formulate a more general and relatively simple model significant effort has recently been done in further development of the microplane model for concrete [3][4][5][6]. Some of the latest results [6] confirm that the model is able to realistically simulate response of concrete structures for arbitrary load histories.

Cracking and damage can principally be modeled in two different ways: (i) discrete (discrete crack model) and (ii) smeared (smeared crack model). The classical local smeared fracture analysis of materials which exhibit softening (quasi-brittle materials) leads in the finite element analysis to the results which are mesh dependent [7]. As well known, the reason for this is the localization of strains in a row of finite elements and a related energy consumption capacity which depends on the element size, i.e. if the finite element mesh is coarse the energy consumption capacity will be larger than when the mesh is fine. Consequently, the model response is mesh dependent. To assure mesh independent results, total energy consumption capacity has to be independent of the element size, i.e. one has to regularize the problem by introducing a so-called localization limiter.

Currently two different approaches are in use. The first one is relatively simple crack band method [8] and the second ones are the so-called higher order methods: Cosserat-continuum [9] and nonlocal continuum approaches of integral type [10][11] or gradient type [9]. Compared to the crack band method the higher order procedures are rather complex, but more general. Mesh independent result can alternatively be obtained by the use of the discrete crack approach [12]. The main drawback of this approach is the need for continuous remeshing, which is a rather complex and time consuming procedure. Moreover, some stress-strain situations (for instance compression) are difficult to model in a discrete sense.

To overcome the problems related to the smeared crack modeling and to avoid complex re-meshing when discrete crack approach is employed a new type of the finite elements has recently been employed [13]. These elements are based on the discontinuous strain field (embedded crack). The background of the method is the discrete crack approach, however, the cracks are here treated at the finite element level with no need for continuous re-meshing. In the concept there are still a number of theoretical difficulties which need to be solved (more than one crack per element, three-dimensionality and other). Presently available numerical examples are restricted to theoretical case studies.
Therefore, a considerable amount of work need to be done before the concept is going to be used as a robust tool for the analysis of structures in engineering practice.

The finite element code for the "every-day" use in engineering practice has to be based on the realistic material model for concrete, i.e. the concrete response should be realistically predicted for arbitrary load history. Moreover, the solution strategies have to be robust, what is from the view point of complexity of the material behaviour not a simple task. In spite of a number of difficulties mentioned above, the smeared crack approach is currently still one of the most general and reliable concept for realistic analysis of concrete and reinforced concrete structures, at least for the practical engineering applications. In the present paper a brief overview of the finite element code which is based on the smeared crack concept and microplane model for concrete is given. On a several numerical examples is demonstrated that the code is able to realistically predict failure mode and resistance of concrete and reinforced concrete structures.

2. Smeared fracture finite element analysis

2.1 General
The finite element (FE) code employed in the present study (MASA) is aimed to be used for the nonlinear smeared fracture analysis of concrete and reinforced concrete structures [14]. It is based on the microplane model and a smeared fracture concept. As regularization procedures the standard or improved crack band approach (stress relaxation method) can be used. Alternatively, the nonlocal integral approach can be employed as well. The concrete is discretized by four-node quadrilateral elements (plane analysis) or by four to eight-node three-dimensional elements. The reinforcement is represented by truss elements. Optionally, it can also be modeled in a smeared way, i.e. smeared inside a row of concrete elements. Besides these standard elements, special linear or nonlinear contact elements are available as well. The analysis is incremental with a standard solution procedure based on the constant stiffness method (explicit approach). The tangent (Newton-Raphson) or secant stiffness approach can also be employed.

2.2 Constitutive law for concrete – microplane material model
In the microplane model is for each integration point of the finite element the material characterized by a relation between the stress and strain components on planes (microplanes) of various, in advance defined, orientations (see Fig. 1). These "monitoring" planes may be imagined to represent the damage planes or weak planes in the microstructure, such as contact layers between aggregates in concrete. In the model the tensorial invariance restrictions need not to be directly enforced. They are automatically satisfied by superimposing the responses from all microplanes in a suitable manner. The basic concept behind the microplane model was advanced in 1938 by G.I. Taylor [15]. Later the model was extended by Bažant and co-workers for modeling of quasi-brittle materials which exhibit softening [3][4][5][6].
The advanced version of the microplane model for concrete was recently proposed by Ožbolt et al. [6]. It is based on the so called relaxed kinematic constraint concept. In the model the microplane (see Figure 1) is defined by its unit normal vector of components \( n_i \). Normal and shear stress and strain components \( (\sigma_N, \sigma_T; \varepsilon_N, \varepsilon_T) \) are considered on each plane. Microplane strains are assumed to be the projections of the macroscopic strain tensor \( \varepsilon_{ij} \) (kinematic constraint). Based on the virtual work approach, the macroscopic stress tensor is obtained as an integral over all microplane orientations:

\[
\sigma_{ij} = \frac{3}{2\pi} \int_{\Omega} \sigma_N n_i n_j d\Omega + \frac{3}{2\pi} \int_{\Omega} \sigma_T (n_i \delta_{rj} + n_j \delta_{ri}) d\Omega
\]  

To realistically model concrete, the normal microplane stress and strain components have to be decomposed into volumetric and deviatoric parts \( (\sigma_N = \sigma_V + \sigma_D, \varepsilon_N = \varepsilon_V + \varepsilon_D; \text{see Figure 1}) \), which leads to the following expression for the macroscopic stress tensor:

\[
\sigma_{ij} = \sigma_V \delta_{ij} + \frac{3}{2\pi} \int_{\Omega} \sigma_D n_i n_j d\Omega + \frac{3}{2\pi} \int_{\Omega} \sigma_T (n_i \delta_{rj} + n_j \delta_{ri}) d\Omega
\]  

For each microplane component, the uniaxial stress-strain relations are calculated as:

\[
\sigma_V = F_V (\varepsilon_{V,\text{eff}}); \quad \sigma_D = F_D (\varepsilon_{D,\text{eff}}); \quad \sigma_{T,\text{r}} = F_{T,\text{r}} (\varepsilon_{T,\text{eff}})
\]

From known macroscopic strain tensor, the microplane strains are calculated based on the kinematic constraint approach. However, in (3) only effective parts of these strains are used to calculate microplane stresses. Finally, the macroscopic stress tensor is obtained from (2). The integration over all microplane directions (21 directions) is performed numerically.

To model concrete cracking for any load history realistically, the effective microplane strains are introduced. They are calculated as:

\[
\varepsilon_{m,\text{eff}} = \varepsilon_m \psi
\]

where subscript \( m \) denotes the corresponding microplane components (\( V, D, T_r \)) and \( \psi \) is a so called discontinuity function. This function accounts for discontinuity of the macroscopic strain field (cracking) on the individual microplanes. It "relaxes" the kinematic constraint which is in the case of strong localization of strains physically unrealistic. Consequently, in the smeared fracture type of the analysis the discontinuity function \( \psi \) enables localization of strains, not only for tensile fracture, but also for dominant compressive type of failure. For more detail see [6].
The model was implemented into the finite element code and a rather broad experience has been gained with it so far. Some of its characteristics are: (1) The main advantage of the model is its conceptual simplicity, i.e. only a set of uniaxial stress-strain curves on the microplane need to be defined and the macroscopic model response comes automatically out as a result of the numerical integration over a number of microplanes; (2) The model covers full three-dimensional range of applicability; (3) It is relatively easy to account for initial anisotropy; (4) The comparison between test data and model response for different stress-strain histories shows good agreement; (5) Unlike to most macroscopic models, in the presented microplane model there is smooth transition from hardening to softening, without unnatural sharp discontinuities. This is very important feature which in the finite element analysis leads to significant reduction of the mesh sensitivity and assure better convergency of the solution; (6) Implementation in the finite element code and a number of numerical studies that have been carried out indicated the capability of the model in realistic prediction of concrete behavior for different stress-strain histories [16].

![Figure 1](image)

**Figure 1.** The concept of the microplane model: a) discretization of the unit volume sphere for each finite element integration point (21 microplane directions) and b) microplane strain components.

2.3 Localization limiter

As mentioned above, to obtain mesh objective results the so-called localization limiter has to be used. In the following, two possible approaches are briefly described – crack band approach and nonlocal integral method.
2.3.1 Crack band method
The main assumption of the crack band method is the localization of damage (crack) into a row of finite elements. To assure a constant and mesh independent energy consumption capacity of concrete (concrete fracture energy $G_f$) the constitutive law needs to be modified such that:

$$G_f = A_f h = \text{const.} \quad (5)$$

where $A_f = \text{area under the uniaxial tensile stress-strain curve}$ and $h = \text{average element size (width of the crack band)}$. Principally, the same relation is valid for uniaxial compression with the assumption that the concrete compressive fracture energy $G_c$ is a material constant:

$$G_c = A_{fc} h = \text{const.} \quad (6)$$

in which $A_{fc} = \text{area under the uniaxial compressive stress-strain curve}$. It is assumed that $G_c$ is approximately 100 times larger than $G_f$ ($G_c \approx 100 G_f$). From (5) and (6), it is obvious that the constitutive law for concrete needs to be adopted to the element size.

Although the crack band method provides results which are independent of the element size, they can still depend on the form and orientation (alignment) of the finite elements. This is especially true when the mesh is relatively coarse or the material is extremely brittle. To reduce this dependency and to keep the simplicity and a relatively low computational effort of the crack band method, a new so called "Stress Relaxation Method" [17] was developed. The method is a combination of the crack band approach and the nonlocal approach of integral type. Due to the low computational costs and, compared to the standard crack band approach, reduced sensitivity to the shape of the mesh the method is appropriate tool for practical applications.

2.3.2 Nonlocal integral approach
The nonlocal integral continuum approach offers a more general possibility to avoid spurious mesh dependency in the smeared fracture analysis of quasi-brittle materials. An effective form of the approach, in which all variables which control the softening are nonlocal and all others are local (nonlocal strain approach), was proposed by Pijaudier-Cabot and Bažant [10]. The key parameter in this approach is the so-called characteristic length $l$. This length controls the size of the representative volume in which the local quantities are averaged.

In the nonlocal continuum concept the stress at a point depends not only on the strain at the same point but also on the strains in the (in advance) defined domain $V$ of the point. In general, the local variable which controls damage needs to be replaced by its nonlocal counterpart obtained by weighted averaging over a spatial neighborhood of each point. If
f(x) is the local variable, which controls the material model response, then the corresponding nonlocal variable is defined as:

$$\tilde{f}(x) = \frac{1}{V_R(x)} \int_V \alpha(x,s) f(s) dV(s) = \int_V \alpha'(x,s) f(s) dV(s)$$  \hspace{1cm} (7)

The bar overscript denotes the averaging operator; \(\alpha(x)\) = nonlocal weighting function; \(x\) and \(s\) are coordinate vectors of the averaging and contributing point, respectively; \(V\) = volume of the entire structure and \(V_R\) = the representative volume calculated as:

$$V_R(x) = \int \alpha(x,s) dV(s) , \hspace{1cm} \alpha'(x,s) = \frac{\alpha(x,s)}{V_R(x)}$$  \hspace{1cm} (8)

The nonlocal weighting function is often taken as the Gauss distribution function:

$$\alpha(r) = \exp\left(-\frac{r^2}{2l^2}\right), \hspace{1cm} r = |s - x|$$  \hspace{1cm} (9)

where \(l\) is the internal (characteristic) length of the nonlocal continuum. Another possible choice is the bell-shaped function:

$$\alpha(r) = \begin{cases} 1 - \left(\frac{r}{R}\right)^2 & 0 \leq r \leq R \\ 0 & R \leq r \end{cases}$$  \hspace{1cm} (10)

where \(R\) (radius) is the parameter related, but not equal to the characteristic length.

The variable, which is to be averaged, must be chosen such that the nonlocal solution exactly agrees with the local solution as long as the material behavior remains in the elastic range. Furthermore, for homogeneous stress-strain field the nonlocal solution must be identical with the local one. Principally, the choice of the variable which is to be averaged is rather arbitrary. However, practically the choice depends on the type of the constitutive law, e.g. plasticity theory, damage theory, microplane model, etc.

It was first assumed that \(l\) is a material property related to the maximum aggregate size \(d_a\) \((l = 3d_a)\). However, it turned out that the characteristic length generally depends not only on the composition of concrete. Namely, it has been found that the optimum value of \(l/d_a\) depends on the stress-strain state. This means that the characteristic length is not a material constant. Therefore, to improve the concept, a new nonlocal approach of integral type which finds the physical background in the interaction of microcracks has
been proposed by Bažant [18] and implemented into a finite element code by Ožbolt and Bažant [11].

In the here presented finite element code the both above mentioned nonlocal approaches are implemented. Theoretically, they are more general than the relatively simple crack band approach. Using these approaches the results of the analysis are mesh independent. However, according to the experience made in the last years there are still a number of problems which make the use of nonlocal approaches in practical applications for concrete and reinforced concrete structures difficult. The results are realistic only for relatively fine meshes. Consequently, the computational costs are often too high and the approach can not practically be used. This is especially true for three-dimensional simulations of reinforced concrete structures.

3. Numerical examples

The discussed finite element code was in the last few years employed in a number of theoretical and practical studies. In the following the capability of the code is demonstrated on several numerical examples. Considered is one rather complex theoretical case of the mixed mode fracture as well as two practical applications, pull-out of a headed stud from a concrete block and shear failure of a slender reinforced concrete beam with and without shear reinforcement. In all case studies the spatial discretization of concrete is performed by 8-node isoparametric finite elements (three-dimensional analysis) or by 4-node elements (axisymmetrical analysis). The reinforcement is modeled by 2-node truss or beam elements assuming an ideally elasto-plastic stress-strain relationship. To assure mesh objective results the crack band approach is employed. As a solution strategy the secant stiffness approach with direct or indirect displacement control was used.

3.1 Double edge notched specimen

The Double-Edge-Notched specimen tested by Nooru-Mohamed [19] was analyzed. The geometry and the test set-up are shown in Figure 2a. The specimen was first loaded by shear load S. Subsequently, at constant shear load, the vertical tensile load T was applied up to failure. The load control procedure was used by moving the upper loading platens in horizontal and vertical direction, respectively. The rotation of the loading platens was restricted. During the application of the horizontal load S, the vertical load was kept zero (T = 0). By subsequent tensile loading the shear force was kept constant. The bottom (support) platens were fixed and, the same as the upper (loading) platens, glued to the surface of the specimen. Three case studies were carried out, i.e. S = 5 kN, S = 10 kN and S = S_{\text{max}}. The finite element discretization is performed by the three-dimensional eight node solid finite elements with eight integration points (see Figure 2b). The width of the finite element model was 5 mm (the actual width of the specimen was 50 mm). The material properties are taken as: Young’s modulus E = 32800 MPa, Poisson’s ratio
\( \nu = 0.2, \) tensile strength \( f_t = 3.0 \text{ MPa}, \) uniaxial compressive strength \( f_c = 38.4 \text{ MPa} \) and concrete fracture energy \( G_F = 0.11 \text{ N/mm}. \)

Figure 2. a) Geometry of the DENS specimen and b) three-dimensional finite element mesh.

Figure 3. Crack patterns observed in the experiment and in the analysis: a) for \( S = 5 \text{ kN} \), b) for \( S = 10 \text{ kN} \) and c) for \( S = S_{\text{max}} \).
As discussed in more detail by Ožbolt and Reinhardt [20], although the results were not optimized they exhibit very good agreement with the experimental results. For illustration the corresponding crack patterns (maximal principal strains) are plotted in Figure 3. The crack patterns obtained in the experiment are shown as well. It can be seen that the present finite element code correctly predicts the crack propagation for mixed-mode fracture, i.e. the calculated and observed crack patterns are for all three load histories almost identical. As shown by di Prisco et al. [21] this was not possible to obtain by using smeared crack models which are based on the tensorial formulation (classical plasticity formulation, nonlocal damage model or gradient plasticity model), especially for the case $S = S_{\text{max}}$.

### 3.2 Pull-out of a headed stud

In practice headed studs are used to transfer loads into concrete members. Extensive experimental and numerical studies have shown that tensile load can be transferred into unreinforced concrete member [22]. If the steel strength of the stud is large enough, the failure of the stud is caused by the formation of a concrete cone. The failure mechanism is controlled by the tensile resistance of concrete, the cracking is stable and therefore the concrete fracture energy can effectively be utilised [23].

![Figure 4. Pull-out of a headed stud: a) Formation of the concrete cone (principal strains, dark areas) analysed by the use of the axisymmetrical version of the code and b) crack pattern observed in the experiment.](image)

For a better understanding of the failure mode as well as to investigate the resistance of the headed stud anchors of various embedment depths, a large number of numerical studies were carried out [22][23]. The numerical simulations based on the axisymmetrical FE-code show good agreement with the test results. Some typical calculated and observed crack patterns for a headed anchor with an embedment depth $h_{\text{ef}} = 450$ mm are shown in Figure 4. As can be seen, the calculated crack pattern agrees well with the crack pattern obtained in the test.

The concrete pull-out failure load of a headed stud relies only on the tensile resistance of concrete. To design safe and economical anchorage it is important to know how the
embedment depth influences the failure capacity of the fastenings (size effect). In Figure 5 the nominal strength of the headed stud ($\sigma_N = P_U/(he_f^2 \cdot \pi)$; $P_U =$ ultimate load) is plotted as a function of the embedment depth. Test results are compared with the numerical results. Furthermore, the prediction by a design formula, which is based on the linear elastic fracture mechanics (CC-method), is plotted as well. Figure 5 indicates that the nominal pull-out capacity decreases with the increase of the embedment depth, i.e. there is a strong size effect on the pull-out capacity. This important effect is correctly predicted by the smeared fracture FE analysis as well as by the proposed design formula.

![Figure 5. Size effect on the concrete cone pull-out load.](image)

The comparison between numerical and test results confirms that the finite element code is able to correctly simulate failure mechanism and ultimate resistance of headed anchors.

### 3.3 Shear failure of reinforced concrete beam

Three-dimensional finite element analysis of reinforced concrete beam loaded in three-point-bending was carried out. The beam failed in the so-called diagonal shear failure mode. Two cases were studied: (a) the beam without shear reinforcement and (b) the beam with shear reinforcement. The geometry of the beam and material properties are shown in Figure 6. The test results for case (b) were obtained after the numerical results were published [24]. The test results for case (a) are not yet available.
Concrete:

$$E = 37272 \text{ MPa}$$
$$v = 0.2$$
$$f_c = 3.9 \text{ MPa}$$
$$f_y = 38.3 \text{ MPa}$$
$$G_f = 110 \text{ J/m}^2$$

Reinforcement:

$$E = 200000 \text{ MPa}$$
$$\sigma_y = 400 \text{ MPa}$$
$$E_T = 3245 \text{ MPa}$$

Figure 6. Calculated and measured load-displacement curves: (a) beam without shear reinforcement and (b) beam with shear reinforcement.

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Figure 7. Calculated and measured load-displacement curves: (a) beam without shear reinforcement and (b) beam with shear reinforcement.
In Figure 7 are shown the calculated and measured (case b) load-displacement curves. The comparison for case (b) shows good agreement. The beam without shear reinforcement fails in diagonal shear failure mode. The crack pattern is shown in Figure 8a (maximum principal strains). At peak load no yielding of reinforcement was observed. The overall behavior of the beam is relatively brittle, what is a typical for this kind of failure mechanism. On the contrary, the beam with stirrups shows rather ductile response (see Fig. 7b). The calculated failure mode after peak load is shown in Figure 8b. The same as in the experiment, the beam fails in the so-called compressive shear failure mode. After ductile response caused by yielding of bending reinforcement, sudden compressive failure took place. Figure 9 shows the strains in the main reinforcement and in the stirrups after peak load. As can be seen, in the main reinforcement as well as in the stirrups there is a localization of strains (yielding). It is obvious that the finite element model is able to account for the activation of the shear reinforcement. Consequently, the failure load compared to the beam without shear reinforcement increases by about 20%, what is in good agreement with design codes.
4. Conclusions

In the present paper some theoretical aspects as well as aspects related to the application of the smeared fracture concept in engineering practice are discussed. In spite of a number of difficulties which can arise when the smeared fracture concept is used in the analysis of concrete and reinforced concrete structures, it is shown that the code which is based on the microplane constitutive law for concrete is able to realistically predict behavior of concrete and reinforced concrete structures. The discussion of the theoretical aspects and the comparison between calculated and test results lead to the following conclusions: (1) The smeared fracture analysis is reliable only if a realistic material model is coupled with an efficient localization limiter; (2) The localization limiter based on a higher order method is computationally expensive and, if the finite element mesh is not fine enough, it may lead to unrealistic results. For most practical problems the crack band method can reasonably well predict behavior of concrete structures; (3) It is demonstrated that the used finite element code is able to realistically predict structural behavior of a rather complex practical cases. From the computational point of view the important feature of the employed constitutive model is the fact that in the model there is no sharp discontinuity which can cause convergency problems and strong mesh dependency; (4) Using a realistic numerical tool a number of phenomena can be explained and, what is important, the amount of expensive experimental work can be reduced.

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6. References


