THE EFFECTS OF FIBER RESTRAINT ON THE SHRINKAGE OF CEMENT COMPOSITES

John Bolander and Zhen Li
University of California at Davis, USA

Abstract

This work describes the development and application of irregular lattice models of cement composites at early ages. An important aspect of the model is the Voronoi diagram discretization of the material domain. The Voronoi diagram defines the lattice element connectivity and the scaling relations used for the element stiffness properties, in a manner that avoids the artificial heterogeneity generally exhibited by irregular lattices. Short fibers are discretely modeled within the three-dimensional domain, directly accounting for length and orientation efficiency of each fiber, as well as wall effects that can bias fiber orientations near the domain boundaries. The mortar phase is subjected to uniform shrinkage and the restraining effects of various fiber contents and orientations are compared with calculations based on elastic shear lag theory.

1. Introduction

One of the primary applications of short fiber reinforcement in cement composites is for reducing problems related to shrinkage of the cement paste [1-3]. The stiff fibers act to reduce shrinkage of the composite material, as they locally restrain the shrinkage of the cement paste [4]. If shrinkage cracks develop, fibers generally bridge the crack surfaces and act to reduce crack opening displacements. The ability of moisture and aggressive chemicals to permeate into cement composites, and thus the durability of such materials, is strongly related to crack opening displacement [5,6].

This paper describes work towards modeling the early age behavior of fiber-reinforced cement composites (FRCC), with particular attention to the restraining effects of the fiber reinforcement on shrinkage. The FRCC is modeled as fiber inclusions within a matrix phase (i.e. mortar or concrete), which is assumed to be homogeneous. A three-dimensional irregular lattice represents the matrix phase of the material. Voronoi scaling of the lattice stiffness coefficients provides an elastically uniform representation of the continuum material. Short fibers are discretely modeled using beam-type elements and their placement in the domain is not constrained by the discretization of the matrix phase. Comparisons with theory serve to validate certain aspects of the three-dimensional irregular lattice model, and set the stage for more general simulations of early age behavior that involve couplings of moisture movement, shrinkage, creep, and fracture [7-10].
2. Irregular lattice modeling of matrix phase

Discretization of the material domain is based on a Voronoi diagram [11], generated from a quasi-random set of seed points. The geometry of the Voronoi diagram defines the connectivity of an irregular lattice, with the seed points acting as lattice sites (or nodes). Figure 1a shows the three-dimensional discretization of a cube, with lattice element \( ij \) partially exposed within the domain interior. This basic element is composed of a zero-size spring set, located at the area centroid (point C) of the Voronoi facet common to nodes \( i \) and \( j \), and rigid arm constraints that link the spring set with the nodal degrees of freedom (Fig. 1b). The spring set consists of three lineal springs, oriented normal and tangential to the facet, and three rotational springs about the same local axes. This notion of a rigid-body-spring model was developed by Kawai [12] and later refined through Voronoi scaling of the spring constants [13,14]. For the lineal springs:

\[
k_x = k_y = k_z = E \frac{A_{ij}}{h_{ij}}
\]

and for the rotational springs:

\[
k_{\phi x} = E \frac{J_p}{h_{ij}}, \quad k_{\phi y} = E \frac{I_{11}}{h_{ij}}, \quad k_{\phi z} = E \frac{I_{22}}{h_{ij}}
\]

where \( A_{ij} \) is the facet area; \( h_{ij} \) is the element length (i.e. the distance between \( i \) and \( j \)); \( E \) is the elastic modulus; \( J_p \) is the polar moment of inertia; and \( I_{11} \) and \( I_{22} \) are the two principal moments of inertia of the facet area. Berton [15] provides additional details on the three-dimensional formulation of this Rigid-Body-Spring Network (RBSN). By virtue of the Voronoi scaling of the spring constants in Eq. 1, the lattice is elastically uniform during uniform straining [13,14]. This quality of the lattice model is important for the simulations of uniform shrinkage presented in section 4.

3. Discrete modeling of short-fiber reinforcement

Each individual fiber is discretely modeled as a series of ordinary beam elements; beam elements are assigned the elastic, plastic, and rupture properties of the fiber. Each beam element has 6 or 12 degrees of freedom, depending on whether the analysis framework is two-
or three-dimensional, respectively. A similar approach has been used to discretely represent fibers in regular lattice models of cement composites [16].

The beam elements (i.e. fibers) are connected to the irregular lattice (i.e. matrix phase) via bond-link elements and rigid arm constraints, as shown in Fig. 2a for the two-dimensional case [13]. There is a bond linkage associated with each Voronoi cell that the fiber trajectory crosses. The bonded length associated with a given spring set is \( l_b \). The stiffness of the tangential bond spring, \( K_t \), is governed by a prescribed bond stress-slip relation that, in the basic case, is characterized by an adhesional strength limit, \( \tau_{au} \), followed by frictional pullout at constant stress, \( \tau_{fu} \) (Fig. 2b). Frictional decay or slip hardening can also be modeled. The same general approach is used in three-dimensions (Fig. 3), where the bond-link element has springs associated with all six generalized displacements. The same bond stress-slip relation is used to control the stiffness of the tangential spring, \( K_t \), representing the interfacial behavior.

Figure 2 – a) Reinforcing component and linkages to irregular lattice; and b) bond stress-slip relation

Figure 3 – Reinforcing component and linkages to three-dimensional irregular lattice
Consider a single fiber embedded within a homogeneous matrix, which is subjected to uniform tension in the \( x \)-direction. Figure 4a shows an exposed view of the single fiber, positioned skew to the direction of loading. For strain \( \varepsilon_m \) in the \( x \)-direction, Fig. 4b shows axial stress in the fiber along its length, where both quantities have been normalized as follows:

\[
\bar{\sigma} = \frac{\sigma}{E_f \varepsilon_m \cos^2 \theta}
\]

and \( \psi \) is the distance from the fiber end divided by the fiber length. \( \sigma \) is the fiber axial stress, \( E_f \) is the elastic modulus of the fiber, and \( \theta \) is the angle between the fiber axis and the loading direction. The solid circles indicate the stress level at the midpoint of each beam element. Both length and orientation efficiency effects are realized by this discrete modeling of the fiber, as would be predicted by elastic shear lag theory [17]. However, the stresses near the fiber ends differ from theory due to the discrete representation of matrix-fiber interaction through a zero-width bond link element. Additional study is needed to improve the correlation between the numerical results and theory near the fiber ends. In related research, precise realization of shear lag and orientation efficiency effects has been accomplished using a semi-discrete modeling of each individual fiber [18,19].

4. Effects of fiber restraint on shrinkage

When modeling FRCC, multiple fibers are positioned in the domain. In this section, the lattice model is used to study the restraining effects of fibers on the shrinkage of cement composites. The lattice model is verified using the micromechanical model of Zhang and Li [4], which is based on elastic shear lag theory and accounts for the random nature of the fiber distribution. The experimental work of Mangat and Azari [20] serves to define the example problem.
In the experimental work [20], both fiber reinforced mortar and concrete shrinkage tests were conducted. Various fiber types and fiber volume fractions were placed within 500x100x100 mm test specimens. The tests involving the largest of the fiber types (i.e. crimped steel fibers) are studied here, since the number of fibers needed to achieve a given volume fraction of fibers is kept to a minimum. Computational expense is strongly dependent on the number of fibers and the resolution of the lattice representing the matrix phase of the composite. The material parameters relevant to simulating the FRCC specimens are: fiber elastic modulus, $E_f = 210$ GPa; fiber length, $l_f = 48.7$ mm; fiber diameter, $d = 1.14$ mm; elastic modulus of the mortar at 28 days, $E_m = 20$ GPa; the volume fraction of coarse aggregate in the concrete specimens, $V_a = 0.0175$; and ultimate shrinkage strain of the mortar, $\varepsilon_{SU} = 825$ microstrain. For direct comparison with the analyses of Zhang and Li [4], the effective fiber length is set to 58.7 mm and the concrete is assumed to shrink uniformly throughout the domain according to

$$\varepsilon_S(t) = \frac{t}{t + 35} \varepsilon_{SU}$$  \hspace{1cm} (4)

where time $t$ is measured in days [21]. Also, the time dependent variation of the mortar elastic modulus is as described in reference 4. The fiber volume fractions considered below are with respect to the mortar content of the concrete mix, which had proportions by weight of 1:2.5:1.2:0.58 (cement : sand : coarse aggregate : water).

Two types of fiber distributions are considered: 1) fibers aligned in the long direction of the specimen (in which shrinkage strain is measured), and 2) randomly oriented fibers. These distributions are shown in Fig. 5 where, for clarity, a fiber volume fraction of 0.1% is shown in each case. Fiber positions and orientations within the domain are determined using a pseudo-random number generator. Fiber placement is not uniformly at random, since fibers can not penetrate the specimen boundaries. More realistic fiber distributions that account for the dynamics of the production process can be determined experimentally or numerically [22], and then used as input to this lattice modeling of FRCC.

Figure 6 shows the shrinkage strains developed in the fiber reinforced concrete over time for two volume fractions of fibers, $V_f = 1\%$ and 2\%. The shrinkage strain prescribed by Eq. 4 is introduced into each lattice element, resulting in uniform shrinkage throughout the domain when no fibers are present. When fibers are included, the individual fibers restrict shrinkage of the concrete, which causes tension in the matrix and compression in the fibers. The difference in modeling shear lag effect at the fiber ends (shown in Fig. 4b) is systematic for all fibers and therefore the numerical model exhibits slightly more shrinkage strain for the case of aligned fibers (compared to the results of Zhang and Li [4]). This difference also affects shrinkage of the random fiber composite, but the effect is offset by directional alignment of fibers near the specimen boundaries. That is, the specimen cross-section constrains the range of fiber orientations when the midpoint of the fiber is within $l_f/2$ from one or more of the boundaries. The numerical model therefore exhibits less shrinkage than is predicted by the theoretical model of the random fiber composite. Agreement with the experimental values is good, considering the simplified representation of shrinkage according to Eq. 4. In actuality, drying shrinkage is highly nonuniform through the cross-section of cement composite members.
Figure 5: Fiber distribution types: a) aligned in axial direction; and b) quasi-random

Figure 6: Shrinkage strain of fiber reinforced concrete: a) $V_f = 1\%$, and b) $V_f = 2\%$ (the experimental data points of Mangat and Azari [20] are obtained from reference 4)
5. Concluding remarks

A novel approach to modeling FRCC has been presented, where fibers exist as discrete entities within a three-dimensional, irregular lattice representation of the material matrix. The model directly accounts for the properties of the matrix and fibers, as well as the length and orientation effects of individual fibers. In this paper, this simulation framework is used to study the precracking restraint provided by fibers during shrinkage of the cement composite. Different fiber distributions and volume fractions are simulated and the results are verified using a micromechanical model [4], which is based on elastic shear lag theory and accounts for the orientation distribution of the fibers. It is clear that the collective, local actions of stiff fibers serve to reduce overall shrinkage of the cement composite. The lattice model is a means for quantitatively linking fundamental micromechanical parameters and shrinkage at the structural scale.

This work is an important step towards more general simulations of the early age behavior and durability mechanics of FRCC, which need to account for moisture movement, nonuniform shrinkage, creep, and fracture within a three-dimensional setting. The discrete modeling of fibers is especially useful when studying the effects of fiber distribution on cracking behavior. Nonuniformity of the fiber distribution can be a primary factor in determining the post-cracking performance of FRCC [23].

6. Acknowledgements

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7. References