Simulation of Expansion in Cement Based Materials Subjected to External Sulfate Attack

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Abstract
Sulfate attack in concrete structures is considered to be among the major durability concerns in civil infrastructure systems. Proper modeling techniques can help us understand the influence of aggressive environments on the concrete performance more readily and accurately. Such an understanding improves the decision making process in every stage of construction and maintenance and will help in better administration of resources. Aspects of cement chemistry, concrete physics, and mechanics are applied to develop a model for predicting sulfate penetration, reaction, damage evolution, and expansion, leading to degradation of cement-based materials exposed to a sulfate solution. The model is refined to address the interaction effects of various parameters using calibration data available from experiments conducted at the National Institutes of Standards and Technology (NIST). Parameters of the model were refined through parametric analysis, consideration of specific boundary conditions, and calibration with experimental data.

Introduction
Portland cement-based materials subjected to attack from sulfates may suffer from two types of damage: loss of strength of the matrix due to degradation of calcium-silicate-hydrate (C-S-H), and volumetric expansion due to formation of gypsum or ettringite that leads to cracking. Loss of strength has been linked to decalcification of the cement paste hydrates upon sulfate ingress, especially C-S-H, while cracking and expansion is attributed to formation of expansive compounds. Efforts of modeling the durability due to external sulfate attack have received attention only in the past decade [1]. An empirical relationship between ettringite formation and expansion is the basis for many models where the expansive strain is linearly related to the concentration of ettringite [2]. This approach has been incorporated in the 4SIGHT program which predicts the durability of concrete structures [3], as well as in a model that calculates the service life of structures subjected to the ingress of sulfates by sorption [4] or mechanical and transport properties [5].

The general conservation-type equations involve diffusion, convection, chemical reaction and sorption, as the governing phenomena for the transfer of mass through concrete. In the case of sulfates, some authors [6] assume that the process is controlled by reaction rather than diffusion, based on an empirical linear equation that links the depth of deterioration at a given time to the tri-calcium silicate (C₃A) content and the concentration of magnesium and sulfate in the original solutions. A solution of the diffusion equation with a term for first order chemical reaction has been proposed to determine the sulfate concentration as a function of time and space [7, 8]. Similar
to the recent work by the NIST group, the diffusion coefficient is represented as a function of the capillary porosity and varies with time since capillary pores fill up with the recently formed minerals [4]. Using micromechanics theory and the diffusion-reaction equation, a model that predicts the expansion of mortar bars [9] has been developed for the 1-D case.

A chemo-mechanical mathematical model has recently been developed to simulate the response of concrete exposed to external sulfate solutions [10,11]. The model is based on the diffusion-reaction moving boundary approach and several mechanisms for the reaction of calcium aluminates with sulfates to form expansive ettringite are considered. A schematic model of the present approach is shown in Figure 1. There are three major input parameters categorized under the main categories of 1) Material Parameters, 2) Exposure & Environmental Loading, and 3) Size & Shape of members. The input parameters are used to estimate physical parameters such as the diffusivity, strength, concentration of available calcium aluminates, and the volumetric proportions due to chemical reactions.

The three distinct but coupled problems of sulfate diffusion, calcium aluminate depletion, and crack front propagation posed above are treated as a moving boundary problem as shown in Figure 2. As the time parameter increases, the sulfates diffuse, and then react with aluminates, resulting in hydration products which expand and potentially cause cracking. The cracking causes the coefficient of diffusivity to change from an uncracked material $D_2$ to a cracked material $D_1$. This change may be linked to a scalar damage parameter that also affects the material stiffness, $E$. This damage parameter, $\omega$, is defined from the available models for uniaxial stress-strain response [10].

![Figure 1. The schematics of the present model for the sulfate attack problem.](image-url)
It is assumed that the calcium aluminates may be a blend of three different phases of tricalcium aluminate, tetracalcium alumino hydrate, and monosulfate with parameter \( \gamma \) representing the proportion of each phase. The cement chemistry notation is used with \( C=CaO, \ S=SiO_2, \ A=Al_2O_3, \ H=H_2O, \) and \( \bar{S}=SO_3 \). The total calcium aluminate phase is then introduced as: \( C_a = \gamma_1 C_4A_3 + \gamma_2 C_4A \bar{S} H_{12} + \gamma_3 CSH_2 \). Each of these compounds may react with the ingressing sulfates (represented in the form of gypsum) according to stoichiometric amounts defined in equations 1-3:

\[
\begin{align*}
C_4A_3 + 3CSH_2 + 14H & \rightarrow C_6A \bar{S}_1 H_{32} + CH \quad (1) \\
C_4A \bar{S} H_{12} + 2CSH_2 + 16H & \rightarrow C_6A \bar{S}_3 H_{32} \quad (2) \\
C_3A + 3CSH_2 + 26H & \rightarrow C_6A \bar{S}_3 H_{32} \quad (3)
\end{align*}
\]

These reactions are lumped in a global sulfate phase-aluminate phase reaction as:

\[
C_a + q\bar{S} \rightarrow C_6A \bar{S}_3 H_{32} \quad (4)
\]

where \( q\bar{S} = (3\gamma_1 + 2\gamma_2 + 3\gamma_3) \) represents the weighted stoichiometric coefficients of the sulfate phase. For any of the individual reactions described above, the volumetric change due to the difference in specific gravity can be calculated using stoichiometric calculations. The schematic representation of sulfate penetration and reaction is shown in Figure 2 as the exposure time increases. The total calcium aluminate phase is divided into reacted and unreacted amounts and represented respectively as \( C_{ar} \) and \( C_{ar}(x,t) \) according to:

\[
C_{ar}(x,t) = C_a - C_{ar}(x,t)
\]  

The coupled differential equations for the depletion of both sulfates and aluminates are solved by means of numerical techniques to take into account the three main effects: limited supply of \( Ca3 \), cracking induced diffusivity, and the degradation effects of the expansive equations [12]. Figure 3 represents the sulfate and reacted \( Ca3 \) concentration profiles as a function of exposure time. The effect of time of reaction on the concentration profiles as predicted by Fick’s second law with a second order reaction can be observed.
A coupled mechanical-diffusion approach is used to relate the chemical reactions to damage evolution. Using a rule-of-mixtures approach, one can relate the expansive nature of the products with the prescribed specific gravity of the compounds. In the present approach, calculations of the volumetric changes between reactants and products were conducted by assuming that ettringite was the only product obtained. Formation of other products such as AFM phase or Gypsum could be also addressed in the volumetric calculations. Once the amount of reacted calcium aluminates into ettringite as a function of time and space are obtained, they can be related to the volumetric strain and the volume changes. It is furthermore assumed that the crystallization pressure of products of reaction results in a bulk expansion of the solid. The constitutive response of the matrix and the expansive stresses are calculated from the imposed volumetric strain. Microcracks are initiated when the strength of the matrix is reached, leading to changes in the diffusivity and a reduction in matrix elastic properties. The variation of diffusivity is linked to the scalar damage parameter due to cracking of the matrix. An averaging scheme is used for all the three phases, and the corresponding expansion based on the molar volumes is defined as:

\[ \epsilon'_V(x,t) = \epsilon^0_V(x,t) - f\Phi = C_{ar} \sum \left( \frac{\Delta V}{V} \right)_p - f\Phi \]  

(6)

According to equation 6, the volumetric strain is adjusted by a shift factor representing the product of the total capillary porosity \( \Phi \) and the parameter \( f \) which is defined as the fraction of capillary porosity available for the dissipation of the expansion products. The magnitude of the shift (delay) in the expansion is due to the amount of capillary porosity. Figure 4 represents the strain generation across a 1/4 portion of the sample exposed on both sides. The model predicts the amount of reacted aluminates, the stresses generated, and internal parameters such as damage, expansion levels, weight gain, stiffness degradation and tensile strength degradation [11].
Applicability of the model

There are many experimental observations that need to be explained in the framework of a mathematical model. The present study focuses on a parametric study of the effect of initial sulfates and the effect of cracked material diffusivity on the expansion characteristics of the sample. The effect of initial sulfate phase concentration on the diffusion and subsequent degradation mechanisms, in addition to the effect of degree of hydration, is studied. The model was calibrated with the experimental data conducted for sulfate attack at NIST.

Effect of Changes in Diffusivity Due to Use of Mineral Admixtures

It is generally accepted that the use of mineral admixtures such as silica fume, flyash or slag significantly improves the sulfate resistance of concrete. This is verified through experimental tests of various researchers [12,13]. A set of parametric studies were conducted to show that by changing the diffusivity of the base material and the extent of cracking induced diffusivity changes, one can address the potential benefits of the supplementary cement based materials.

A variable diffusivity parameter was introduced in order to reflect the use of mineral admixtures in changing the microstructure. Test data comparing samples with two different initial diffusivities of \( D_2 = 1 \times 10^{-12} \text{ m}^2/\text{s} \) and \( 1 \times 10^{-13} \text{ m}^2/\text{s} \) were considered in Figures 5a and 5b. In each of these trial runs, a range of diffusivity changes due to cracking was also considered spanning three orders of magnitude. A parameter \( D_1/D_2 = 10 \) would indicate that the cracked material would increase its diffusivity by ten fold as compared to the uncracked base material. Results are shown in Figures 5a and 5b and indicate that as the time increases, an increased diffusivity due to cracking would directly result in faster rates of expansion. The expansion and degradation would take place faster by increasing the \( D_1/D_2 \) ratio. This response also happens much faster for the specimen with a higher initial diffusivity \( (D_2 = 1 \times 10^{-12} \text{ m}^2/\text{s}) \). Analysis of various experimental results indicates that the choice of the range of values for \( D_1 \) (undamaged material diffusivity) is consistent with the water/cement mass ratio and microstructure of the mix designs used. When all other parameters

Figure 4. Strain generation across a 1/4 portion of the sample exposed on both sides
were kept the same, lower values of w/c and the use of pozzolanic materials lead to a lower diffusivity, and hence to a slower reaction, and expansion rate. This mechanism is clearly predicted by the model. When the uncracked diffusivity is reduced by an order of magnitude from $1 \times 10^{-12}$ m$^2$/s to $1 \times 10^{-15}$ m$^2$/s the period to achieve the same level of expansion is significantly extended as well. This time factor depends on the level of damage caused by the cracking as well. Note that the higher the level of damage due to cracking as measured by $D_1/D_2$ ratio, the faster the degradation process. The linear strain measure was used to represent the expansion term here. It was assumed that the linear expansion component would be 1/3 of the volumetric strain under the conditions that the strain is homogeneous and isotropic.

![Graph A](image1.png)

![Graph B](image2.png)

Figure 5 Parametric study of the effect of diffusivity of the cracked material on the expansion-time curves for specimens with various levels of damage due to cracking.
Figure 6 Effect of initial gypsum content on the parametric study of the expansion time curves for specimens containing two levels of initial \( C_3A \), A) \( C_3A = 6\% \), and B) \( C_3A = 10\% \).

Effect of initial gypsum content on the expansion-time curves for specimens containing two levels of initial \( C_3A \), 6 \% and 10 \%, are presented in Figures 6a, and 6b. Note that in both cases, increase in the amount of initial gypsum increases the potential for damage and degradation. It is
interesting to note that the rate of expansion is different in different samples as well. In specimens containing 10% initial C₃A, even with only the use of 2% initial sulfates, significant damage is observed after 120 days (0.6%); whereas with the same initial sulfates content, a specimen with 6% initial C₃A would not show a significant expansion within that same period.

![Diagram showing expansion-time curves](image)

Figure 7 Effect of specimen age when testing begins on the expansion-time curves.

The effect of curing prior to testing is studied in Figure 7. These tests used a blend of cements with levels of C₃A content ranging from 2% to 11.6%. The w/c ratio used was 0.485, with a sand/cement ratio of 2.75. Length change of mortar bars 25 mm x 25 mm x 275 mm (1 in x 1 in x 11 in) in dimensions were recorded on a weekly basis for periods of up to three years. For the present evaluation, two samples are compared with one day and one year of curing prior to start of the test. To properly define the microstructure in samples, which were cured for one year prior to immersion in sulfates, a higher degree of hydration in addition to a lower degree of porosity was considered. For this reason, the parameters of degree of hydration and internal porosity were assumed to be at 0.9 and 0.25 for the 1 d cured specimen as compared to 0.95 and 0.15 for the one year cured specimen. Both the samples contained the same level of initial C₃A and initial sulfates. Note that according to Figure 7, a good correlation is observed between the experimental and simulation responses. It is interesting to note that according to the present model, the fact that the samples with higher internal porosities are able to be more resistant to the sulfate attack is shown. The samples which had year of hydration were showing significant levels of expansion after 40 d, whereas samples subjected to curing to only one day had a sufficiently high level of internal porosity and the expansion started to be detectable after almost 90 d of immersion. This should by no means be used as a justification to shorten the curing period of the concrete; it is only intended to show that the model is capable to incorporate some of the experimental observations. At the same time, it must be noted that as we move toward high performance and denser concrete systems, the additional gain in the reduction of diffusivity parameter may be offset by the lack of internal porosity, which allows for the dissipation of internal pressures caused by ettringite formation.
Effect of initial C₃A content

The resistance of plain and blended concrete (C₃A content between 5% - 12%) to sulfate solutions has been studied by several authors [14]. Effect of initial C₃A content was studied by evaluating the test results of a series of experiments conducted by Ferraris et al. [15]. A reasonable fit is obtained by adjusting the input data in this case. The results are shown in Figures 8a, and b. The parameters used in the theoretical calculations are also shown. The values of the parameters for fitting the data are reported as a degree of hydration of 0.9, and an initial sulfate content equivalent to 6% was assumed. Note that as the initial C₃A increases, the expansion also increases as a function of time. No attempt was made to fit the expansion curve results with the theoretical model and only the trends of the experimental data are simulated using expected values of material parameters. Figure 8b shows a similar set of data obtained for specimens with an initial C₃A at 11.6%. Note that the rate of expansion in these samples is significantly higher than the previous case, and the simulation graphs are able to address this enhanced expansion activity quite well. Due to the large magnitude of expansive forces expected in this case, the amount of residual stress assumed in this case was set at 10 MPa, as compared to the previous case, which was assumed at 5 MPa.

Conclusions

Applications of a theoretical simulation model to predict the degradation due to external sulfate attack on cement-based materials are discussed. Simulations of the model using a series of parametric studies indicate that the effects of diffusivity of the cracked and uncracked material can play a significant role in the characteristics of sulfate penetration. The amount of initial sulfates in the Portland cement is also an important parameter as predicted by the model. Parameters of the model are chosen to be compatible with the actual mix design of the materials. Effect of initial C₃A content and the age of specimen at time of testing were shown to be predicted favorably by the model parameters.

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Figure 8 Effect of initial C\textsubscript{3}A content on the expansion time response of specimens.

**References**


