MULTISCALE FRAMEWORK TO MODEL FIBRE REINFORCED CEMENTITIOUS COMPOSITE AND STUDY ITS MICROSTRUCTURE

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Abstract

A number of different high performance cementitious materials have been recently developed. One class of materials is fibre reinforced cementitious composites (FRCCs). In this study we present a computational two-scale approach to model FRCCs. The main focus is on the discrete fibre distribution on the meso-scale. In our model fibres are added to a continuum damage model in a discrete way, and their influence is taken into account by super-imposing pull-out forces on the matrix material. These fibre pull-out forces represent the link between meso- and micro-scale and are described by an analytical micro-mechanical based model. Computational efficiency is enhanced since the fibres are not explicitly modelled.

1. INTRODUCTION

For use and design of fibre reinforced cementitious composites (FRCCs) computational tools are needed to simulate the influence of material structure on the resulting mechanical properties.

Concrete is a very brittle material with a low tensile strength. By adding fibres the tensile strength of concrete can be increased while reducing the brittleness in the postpeak regime. In some cases even a kind of hardening behaviour can be obtained [1] and the crack pattern can be described as distributed in contrast to localised cracking which is typical of plain concrete.

Our goal is to develop a numerical tool that helps to understand how micro-structural aspects influence the behaviour of the material. In particular, we are interested in the fibre distribution in a sample and its effect on the mechanical properties of FRCC. Apparently, shape and production of a specimen influence distribution and orientation of fibres. Our model helps to investigate the impact of these characteristics on the material behaviour.

Existing approaches to model FRCCs range from analytical to computational and from micro- to macro-scale models. On the macro-scale, constitutive models have been developed that can represent the behaviour of FRCCs in a smeared way [2]. On the meso-scale the fibre distribution is treated explicitly. Lattice models provide a natural way to model discrete fibres in a matrix material [3]. To be able to use continuum models to describe the matrix, fibres have been represented by trusses [4]. By using partition of unity based techniques, such as the extended finite element method, the structure of fibre reinforced material can be modelled without discretising the geometry of each single fibre [5]. On the micro-scale an analytical
approach has been used to describe and develop a class of high performance fibre reinforced cementitious composites called engineered cementitious composites (ECC) [1,6]. In multiscale models the material is treated on macro-, meso- and micro-scale [7].

Our computational approach focuses on the meso-scale. It makes use of information from the micro-scale by including a micro mechanical model describing fibre pull-out [6].

2. MODEL DESCRIPTION AND MAIN ASSUMPTIONS

The behaviour of a composite material depends on its structure, on the properties of each component, their interaction and distribution. Our model is based on the assumption that two processes govern the material behaviour of FRCCs: damage development in the cement matrix and interaction between fibre and matrix. The fibre-matrix interaction takes place mainly on the micro-scale. Since it is difficult to measure it directly, we assume that it can be represented by the single fibre pull-out behaviour, which is described by a relation between a fibre pull-out force and pull-out distance. This assumption is reasonable for materials containing a small volume fraction of fibres as for example ECCs. It does not hold for high volume fractions of fibres as used in slurry infiltrated fibre concrete (SIFCON). The cement matrix is described by a local, isotropic damage model with exponential softening. To obtain discretisation independent results a fracture-energy regularisation scheme is employed.

Figure 1: Mechanical model of FRCC

Figure 1 shows the mechanical model describing FRCCs. A domain $\Omega$ of a brittle matrix material is considered. At parts of the domain tensile forces $F$ are applied. The domain contains an initial crack, which is bridged by a fibre. When the crack opens it is assumed that one end of the fibre is pulled out of the matrix. The fibres are physically neglected in the model. Instead of modelling the fibre, only the reaction forces acting on the matrix are considered. They are assumed to be equal to the forces that can be measured during a single
fibre pull-out experiment. The relation between the fibre pull-out force $P$ and the fibre pull-out distance $\Delta$ is described by a micro-mechanical model [6]. We assume that the pull-out distance $\Delta$ is equal to the crack opening at the intersection point between fibre and crack.

To solve the above problem the finite element method is used. Since the fibres are represented by the fibre forces there is no need to discretise them. To simplify the mesh generation and to preserve mesh independent results the fibre forces are mapped to the nodes that surround the fibre end-points within a specified radius.

3. MATHEMATICAL FORMULATION

In the mathematical formulation a domain $\Omega$ is considered that is split by a crack into a domain $\Omega^+$ and $\Omega^-$. A fibre bridges the crack. The fibre is represented by the forces acting between fibre and matrix. They are named $p^+$ or $p^-$ depending on the side of the crack and are distributed over a certain area around the fibre endpoints $\Omega_{\text{fib}}^+$ and $\Omega_{\text{fib}}^-$, respectively. The whole domain is loaded by an external force $f_{\text{ext}}$, which is distributed over an area $\Omega_f$.

The virtual work equation for the system can be expressed as

$$\frac{\partial W_{\text{int}}}{\partial \delta} = \frac{\partial W_{\text{ext}}}{\partial \delta} + \frac{\partial W_{\text{fib}}}{\partial \delta},$$

in which $\partial W_{\text{int}}$ describes the matrix, $\partial W_{\text{fib}}$ represents the action of the fibre on the matrix and $\partial W_{\text{ext}}$ follows from the applied external load. Expansion of the terms in (1) yields:

$$\tilde{\mathbf{f}}: \varepsilon d\Omega = \tilde{\mathbf{f}}_{\text{ext}}: \delta \mathbf{u} d\Omega_f + \tilde{\mathbf{a}}_{\text{fib}} \tilde{\mathbf{n}}^+ (\Delta) \cdot \delta \mathbf{u} d\Omega_{\text{fib}}^+ + \tilde{\mathbf{n}}^- (\Delta) \cdot \delta \mathbf{u} d\Omega_{\text{fib}}^-,$$

where $\mathbf{\sigma}$ is the Cauchy stress tensor, $\varepsilon$ is the linear strain tensor, $f_{\text{ext}}$ is the external load vector, $\mathbf{u}$ is the displacement vector, $n_{\text{fib}}$ is the number of fibres in the system and $\Delta$ is the fibre pull-out distance.

The matrix material is described by a simple isotropic damage model in which

$$\mathbf{\sigma} = (1 - \omega) \mathbf{D} : \varepsilon,$$

where $\omega$ is a damage variable ranging from zero to one and $\mathbf{D}$ is the elastic material stiffness tensor. Softening is described by an exponential softening law [8]

$$\omega = \begin{cases} \tilde{\epsilon}_0 & \kappa < \epsilon_0 \\ \frac{1 - \epsilon_0}{\epsilon_0} e^{-\kappa - \epsilon_0} & \kappa \geq \epsilon_0 \end{cases},$$

in which $\kappa$ is the history variable related to damage evolution and $\epsilon_0$ specifies the slope of the softening branch. The damage initiation strain $\epsilon_0$ is defined as
\[ \varepsilon_0 = \frac{f_t}{E}, \quad (5) \]

where \( f_t \) is the tensile strength of the cement matrix and \( E \) its Young’s modulus. For the equivalent strain \( \bar{\varepsilon} \) an energy formulation is used

\[ \bar{\varepsilon} = \sqrt{\varepsilon : D : \varepsilon} \quad (6) \]

To provide mesh objectivity of the solution a fracture energy regularisation is employed [9]. The softening parameter is thus modified as follows:

\[ \varepsilon_i = \frac{\lambda}{h} \delta_\varepsilon - \frac{\varepsilon_0}{2} \frac{\delta_\varepsilon}{E} - \frac{\varepsilon_0}{2}, \quad (7) \]

in which \( \lambda \) denotes the width of the damage zone and \( h \) is an equivalent element size. Using triangular elements the equivalent element size \( h \) is chosen to be the radius of the in-circle of the triangle. While the fracture energy regularisation approach does neither lead to an objective width of the damage zone nor to a fully discretisation independent damage path, it ensures mesh independent energy dissipation. Together with its computational efficiency this is sufficient for using it with the computational strategy to include fibres.

The fibre force \( p^{+/-} \) is split into the scalar value of the pull-out force \( P \), a function \( f(x) \), which distributes the fibre pull-out force linearly over the area around the fibre endpoint \( \Omega_{\text{fib}} \), and a direction vector \( \mathbf{n}_{\text{fib}}^{+/−} \). The direction vector can be different for each fibre end. Using these definitions, reads:

\[ \tilde{\n}^+ P(\Delta) \cdot \delta \mathbf{u} d\Omega_{\text{fib}}^+ = \tilde{\n}^+ P(\Delta) f(x) \mathbf{n}_{\text{fib}}^+ \cdot \delta \mathbf{u} d\Omega_{\text{fib}}^+. \quad (8) \]

The fibre pull-out force represents the link between meso- and micro-scale. During the fibre loading it is described by a micro mechanical based model [6]. To simplify matters, only the debonding range is treated here. Since the original function is non-smooth, which may lead to numerical difficulties, it is partially smoothed by pre-multiplying the second term with \( \tanh(a\Delta) \). The numerical parameter \( a \) can be freely chosen according to the appropriate slope. The fibre pull-out relation can be written as follows:

\[ P(\Delta) = \sqrt{\frac{\pi^2 \tau_0 E_{\text{fib}} d_{\text{fib}}^3 (1+\eta)}{2} \Delta + \tanh(a\Delta) \frac{\pi^2 G_d E_{\text{fib}} d_{\text{fib}}^3}{2}}, \quad \text{with} \quad \eta = \frac{E_{\text{fib}} V_{\text{fib}}}{E_m V_m}, \quad (9) \]

in which \( E_{\text{fib}} \) is the Young’s modulus of the fibre, \( V_{\text{fib}} \) is the volume fraction of the fibre, \( E_m \) and \( V_m \) are the Young’s modulus and the volume fraction of the matrix. The frictional stress on the debonded interface is given by \( \tau_0 \), \( d_{\text{fib}} \) is the fibre diameter and \( G_d \) is the chemical bond strength between fibre and matrix. Secant unloading is used.

The pull-out distance \( \Delta \) is assumed to be the same as the opening of the crack, which is bridged by the fibre, at the intersection of fibre and crack. This assumption provides the link
between the fibre and the matrix terms. Since in this study a continuum damage model is used to describe the cement matrix, the crack opening is not directly computed. To calculate the pull-out distance the following relationship is used:

\[ \Delta = \omega_c \left\| u_1 - u_2 \right\| - \omega_c \tilde{u}_f, \]  

(10)

where \( u_1 \) and \( u_2 \) are defined as the displacements of the matrix at the location of the fibre endpoints, \( \tilde{u}_f \) is the fibre elongation, and \( \omega_c \) is the damage value of the matrix at the intersection between fibre and crack. In practice, the highest damage value of the matrix along the fibre is chosen. Using the product of the elongation of the fibre and the bridged damage means that, if the fibre is bridging a fully damaged element, the whole elongation between the fibre endpoints is assumed to be the fibre pull-out. Without any damage the pull-out distance is zero.

In the following examples a staggered solution scheme is employed. Within one loading step, the damage problem is solved and based on this solution the fibre forces are computed. They are applied to the system and the damage problem is solved again. This process is repeated until equilibrium is reached. Then the next loading step is computed.

4. EXAMPLES

First a simple test is presented to investigate the basic properties of the model. Then a comparison between two different fibre distributions is shown to illustrate the potential use of the approach.

4.1 Two-fibre example

A tension test is simulated. The system is shown in Figure 2. A bar consisting of ten linear triangular elements is clamped at the left end and pulled at the right end. The specimen is 10 mm long and 1 mm high. For the cement matrix the following set of parameters is used: Young’s modulus \( E = 36800 \text{ N/mm}^2 \), Poisson’s ratio \( \nu = 0.25 \), tensile strength \( f_t = 1 \text{ N/mm}^2 \), softening parameter \( \epsilon_s = 0.0025 \) and localisation zone width \( \lambda = 0.1 \text{ mm} \). In the fibre pull-out relation the following parameters are employed: fibre Young’s modulus \( E_f = 42800 \text{ N/mm}^2 \), chemical bond strength \( G_d = 0.0016 \text{ Nmm/mm}^2 \), frictional stress \( \tau_s = 1.1 \text{ N/mm}^2 \), slip hardening parameter \( \beta = 1.15 \), numerical parameter \( a = 100 \), fibre diameter \( d_f = 0.039 \text{ mm} \) and fibre length \( l_f = 5.8 \text{ mm} \). The fibre forces are distributed to the matrix via a half circle with a radius of \( r = 0.21 l_f \) around each fibre endpoint.

To initialise damage the strength of the middle elements is reduced by a factor 0.5 (dark grey section in the left part of Figure 2) and the strength of the elements at the right end are reduced by a factor 0.8 (light grey area).

Figure 2: Test setup, mechanical system (left) and fibres (right)
The fibres are positioned symmetrically in the sample as shown in the right part of Figure 2. To keep the system simple and to reach the effect of multiple cracking, five fibres are located on top of each other at each of the two indicated fibre positions.

The comparison of a pure matrix sample with a sample including fibres shows the expected results. In the left part of Figure 3 the matrix sample is depicted. In the right part of the figure the damage plots of the fibre sample are presented. The damage initiation starts in both samples in the weak centre elements. In the matrix specimen the damage localises in the centre elements. In the fibre specimen the damage grows only initially in the centre elements. The fibres transmit forces to the ends of the bar. The damage growth in the centre elements is arrested and damage starts to localise at the weaker sample end.

![Figure 3: Damage plots: on the left side a pure matrix sample and on the right side a sample including fibres](image)

The force-displacement plots of the fibre, the matrix sample and the fibre forces are depicted in Figure 4. The fibre sample reaches a higher peak strength than the matrix sample. The fibres are activated as soon as damage starts to grow in the centre elements, which are bridged by the fibres. While the matrix sample is already softening the fibre specimen can still be loaded. When the strength of the matrix at the right end of the bar is reached, softening starts in the fibre sample as well. The damage localises at the sample end and the fibres are unloaded. In the right part of Figure 4 the fibre forces are plotted in terms of the fibre pull-out distance $\Delta$. The nonlinear loading and the linear unloading phase can be clearly distinguished.

![Figure 4: Force displacement (left) and fibre force (right) plots of the matrix and the fibre sample](image)
4.2 Twenty-fibre example

In the second example a larger problem is analysed. A tension test is performed with a notched bar including two arbitrary distributions of twenty fibres (see Figure 5). An area of 1 mm width at the ends of the bar is clamped so that fibres can enter the supports. The specimen has a length of 10 mm and a width of 2 mm while the fibres have a length of 2 mm. The numerical parameter \( a \) is set to \( a = 100000 \). Because of the notches no weak elements are needed for crack initiation. Apart from that, the same parameters as in the first example are used.

Figure 5: Mechanical system (top left), discretisation (top right), fibre distribution 1 (bottom right) and fibre distribution 2 (bottom left)

Figure 6: Force-displacement plots of the matrix sample in comparison with the two fibre samples

Figure 7: Damage plots of a pure matrix specimen (left column), fibre distribution 1 (middle column) and fibre distribution 2 (right column)
Comparing the force displacement plots of the two fibre samples and a reference matrix sample without fibres shows (Figure 6) that the addition of fibres increases the strength and leads to a more ductile material. Both fibre samples reach a comparable strength and show a similar behaviour. However, their damage behaviour shows remarkable differences. In Figure 7 damage patterns related to the matrix sample, fibre distribution 1 and 2 are reported. The damage initialises in all samples at the notches (first row). In the matrix sample the damage localises after further loading between the notches. Distribution 2 shows a similar behaviour (right column) although the damage grows in a circle around the fibre endpoints. In distribution 1 two parallel damage zones develop around the notches.

5. CONCLUSIONS

A physically realistic sequential two scale approach to model FRCCs has been presented. The model includes discrete fibre distributions in an efficient way into a continuum damage approach. Fibres are not explicitly discretised. This simplifies the process of the mesh generation. Since a number of nonlinear processes, different cracks and the active fibres, are competing, convergence may be in some cases difficult. Nevertheless, the approach can represent the influence of a certain fibre distribution on the overall mechanical behaviour and especially on the damage pattern of the sample. This allows the investigation of the interaction between specimen size and geometry and fibre length and distribution.

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