MODELING OF EARLY AGE BEHAVIOR OF UNDERGROUND STRUCTURE

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Abstract
At early ages, concrete cast-in-place tends to crack owing to transportation of heat and moisture. Due to the underground water and complex boundary during construction, the crack control of underground structure poses an important role in early age. This paper deals with the concrete crack prediction by analyzing the moisture, temperature and stress distribution. To predict the response of a structural member in the early period of construction with age, a step-by-step method is necessary. At each step, deformation due to thermal variation, drying shrinkage, and creep during current time interval is imposed. The construction process is simulated by FEM for cracking prediction, and the time-dependent factors are taken into account, such as boundary and concrete strength. This paper takes a project in Shanghai for example to apply above method. The critical position and crack width are predicted. Finally, corresponding advices for structure design and for construction are given to prevent early age cracking.

1. INTRODUCTION
With rapid economical development of China, more and more large-scale cast-in-situ underground structures are applied to infrastructures. The chemical processes that occur during cement hydration in the first few days after casting are accompanied by a significant increase of temperature and a volume change. However, because the stiffness of the concrete mass at this stage is relatively low, a change in volume results in only moderate stresses. As time passes, the hydration process continues and the stiffness of concrete increases markedly. During this hardening phase, if ambient temperature drops, significant tensile stress may develop. Once the stress surpasses concrete tensile strength, the cracks begin to develop. For prevention from seepage of groundwater, the cracking control poses an important role in underground structures. Thermal and mechanical boundaries of underground structures are
more complex, and the structure styles are quite different from the surface structures. Therefore, the development of early age crack in underground structures is also quite special. In this present paper, a brief outline of 3D thermal and stress analysis procedures is given, which assumes that material enters directly from an elastic state to a failure state. The generation and dissipation of heat within the concrete is accounted for, including the effect of ambient temperature. The stress-deformation analysis procedures include separate treatment of temperature-induced, shrinkage and creep deformations. Development of concrete properties is expressed as a time-dependent function based on experimentally obtained data, such as Young’s modulus, tensile strength. It is assumed that failure occurs when the maximum (normal) principal stress reaches either the uniaxial tension strength, or the uniaxial compression strength. Finally, a project in Shanghai is taken for an example to apply above method.

2. ANALYSIS METHOD

2.1 Thermal Analysis

After concrete is placed in the field, it will be sustained in a thermal environment different from mixing. At early age, a great deal of heat will be generated with the hydration of cement. With day after night, temperature would be varied with time. With the help of heat conduction theory, heat diffusion within concrete is easily expressed in a differential equation (Eq. (1)):

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{f(t)}{c \gamma} \frac{\partial T}{\partial t}$$

where $T$ is temperature of concrete (K), $k_x$, $k_y$, and $k_z$ are the diffusion ratios along the three different directions, $c$ is the specific heat ratio (J/kg K) and $\gamma$ is the density of concrete (kg/m³). For simplicity, it was reasonably assumed that $k_x$, $k_y$, and $k_z$ are equal and constant. The function $f(t)$ describes heat generation from hydration, which can be simply expressed as follows (Eqs. (2) and (3))

$$f(t) = \frac{\partial F(t)}{\partial t} = mWQ_0 \exp(-mt)$$

and

$$F(t) = WQ_0(1 - \exp(-mt))$$

where $W$ is the cement content per unit volume (kg/m³), $Q_0$ is the heat per unit cement content (J/kg), $m$ is the hydration rate.

Boundary conditions around a given structure may be summarized with a simplified form (Eqs. (3) and (4)):

At boundary $S_1$: $T(x,y,z,t) = T_0 = \text{const}$
At boundary S2: \[ k_x \frac{\partial T}{\partial x} l_x + k_y \frac{\partial T}{\partial y} l_y + k_z \frac{\partial T}{\partial z} l_z + \alpha \cdot (T - T_m) = 0 \] (5)

where \( \alpha \) is the relative diffusion ratio, which is determined by boundary types, curing conditions and other factors.

### 2.2 Stress Analysis

Once the age-dependent temperature field \( T \) is calculated from thermal analysis, stress analysis may be carried out to compute the stress and strain evolution. To predict the response of structure member in the early period of construction, a step-by-step method is necessary. At the beginning of each time step, deformation due to thermal variation (hydration and environment), drying shrinkage and creep during the current time interval is imposed. This imposed incremental strain on any point at \( \text{ith} \) time interval is defined as (Eq. (6))

\[
\Delta \varepsilon_{i}^{\text{tot}} = \Delta \varepsilon_{i}^{\text{s}} + \Delta \varepsilon_{i}^{\text{t}} + \Delta \varepsilon_{i}^{\text{c}}
\] (6)

where, \( \Delta \varepsilon_{i}^{\text{s}} \) is the incremental shrinkage strain at time \( t_i \), \( \Delta \varepsilon_{i}^{\text{t}} \) is the incremental thermal strain, \( \Delta \varepsilon_{i}^{\text{c}} \) is the incremental creep strain, and the \( \text{i} \)th interval is the one between setting time \( (i)t \) and \( (i+1)t \).

Mechanism of shrinkage is far more complicated. Empirical equations have been widely used in numerical calculation. In this paper, the shrinkage strain can be described as\[2\]:

\[
\varepsilon_{y}(t) = \varepsilon_{y}^{0} \cdot M_{1} \cdot M_{2} \cdot \ldots \cdot M_{n} (1 - e^{-bt})
\] (7)

where \( \varepsilon_{y}(t) \) is the shrinkage strain at time \( t \), \( \varepsilon_{y}^{0} \) is the ultimate strain, \( M_{1}, M_{2}, \ldots, M_{n} \) are kinds of factors.

A complete stable incremental analysis was proposed by Bazant [3] with the assumption that \{\( ds \)/dt and \( E_{i}(t_{0}) \)\} are constant within each time interval \( (t_{i-1} < t < t_{i}) \). The incremental stress–strain law becomes (Eqs. (8)–(12))

\[
\{\Delta \sigma\}_{r} = E_{r}^{*} [\hat{D}](\{\Delta \varepsilon\}_{r} - \{\Delta \eta\}_{r-1})
\] (8)

Where

\[
\frac{1}{E_{r}^{*}} = \frac{1}{E_{r-0.5}} + \hat{a} \sum_{i=1}^{n} \frac{1 - [1 - \exp(-\Delta \tau_{r}/\tau_{i})] \tau_{i}/\Delta \tau_{r}}{(\hat{E}_{r})_{r-0.5}}
\] (9)

\[
\{\Delta \eta\}_{r} = \hat{a} \sum_{i=1}^{n} \frac{[1 - \exp(\Delta \tau_{r}/\tau_{i})] \{\varepsilon_{i}^{*}\}_{r-1}}{\{\varepsilon_{i}^{*}\}_{r-1}}
\] (10)

The effect of prior stress histories is contained in the set of vectors \( \{\varepsilon_{i}^{*}\} \):
\[ \{e^*_i\}_r = \frac{\Delta \sigma_i [1 - \exp(-\Delta t/\tau_i)] \tau_i/\Delta t_i + \{e^*_i\}_{r-1} \exp(-\Delta t_i/\tau_i)}{(E_i)_{r-0.5}} \]  

where the coefficients \( E_i(t') \) are restricted to the form

\[ \frac{1}{E_i(t')} = a_i + b_i(t')^n_i \]

where \( a_i, b_i \) and \( n_i \) are material constants.

To consider all the age-dependent factors consistently and simply, we suggested the empirical model of development in cement hydration, which was utilized to define and describe other aging properties, such as strength, elastic modulus, etc. The hydration rate was defined as follows (Eqs. (13)–(15)):

\[ H(t) = \frac{t}{C + Dt} \]
\[ f_i(t) = H(t)f_i(28) \]
\[ E_i(t) = \sqrt{H(t)E_i(28)} \]

where \( t \) is current age (in days), and \( f_i(28) \) and \( E_i(28) \) are the 28-day strength and elasticity modulus, respectively.

2.3 Failure Criteria

The failure of concrete under 3D stress state can be very complicated. In this paper, the Rankin failure criteria is applied, which can be described as:

\[ f = \max\left( |\sigma_1|, |\sigma_2|, |\sigma_3| \right) - f_i \leq 0 \]

![Fig1: Max principal stress yield surface](image)
3. PROJECT APPLICATION

3.1 Project Introduction

500kV JingAn transformer substation project is constructed for the Shanghai World Expo 2010, which is in the Jingan District of Shanghai. The extent of the transformer substation is 220 meters. The structure of the project is a four-floor concrete cylinder, 130m in diameter, 34m in excavation depth. With regard to the important role in World Expo and high level of underground water in Shanghai, it is necessary to evaluate its risk of cracking in early age.

![Figure 2: The profile of Jingan transformer substation](image)

This paper takes the Level 0 to example to model the early age behaviors in underground structure. Level 0 is compose of slabs and beams with different scales. The thickness of slab is 300mm, and height of beam is 2000mm. In view of effects of sunshine and ambient air convention, the concrete slab with huge surface area is easy to dry shrink. Besides, the beams and columns compose a complex constraint system on slabs. Therefore, the cracking risk of slab in early age exists

3.2 Analysis Results

With regard to that the height of beams is much larger than the thickness of slabs, the early age behaviours of beam and slab are respectively analyzed. The constraints of columns on beam are expressed as only the mechanical boundary of beam, and the constraints of slab on beam are neglected. When the slab is simulated, the constraints of columns and beams are both considered. The finite-element calculated results are showed in from Fig (3) to Fig (6).
Figure 3: The normal stress in the beam

Figure 4: The principal stress in the beam during the early age

Figure 5: The normal stress in the slab

Figure 6: The principal stress in the slab during the early age
3.3 Discussion and construction Advices
Cross section of the beams in Level 0 is very large, which may cause the great temperature difference between inner and outer surface. With the development of temperature difference, the markedly tensile stress occurs in the middle of the beam surface, which can induce the early age surface cracks. Therefore, effective control temperature difference between inner and outer surface is the main measure to control early age cracks on the beam. The slab is thin compared with the beam, but it has relatively large specific surface. Therefore, the dry shrinkage is the main reason of early age cracking. The beams, the columns and the templates compose a 3D constraint system on the slab. Above analysis shows that marked tensile stress occurs at the corner of slab, where slab is constrained in two directions. To sum up, for preventing thermal cracking in the beams and shrinkage cracking in the slab, the measures to remain the heat and moisture of young concrete are necessary. The maximum principal stresses of slab and beams both surpass the ultimate strength of concrete according with above analysis results. In the practical construction, because of proper maintain measures in early age, no macro crack occurs in the structure.

4. CONCLUDING REMARKS
This paper applies a 3D FEM numerical method to assess the early-age cracking risk of underground structure. The temperature field and stress field are both analyzed in the early age. According to this study, the following general results are concluded:
• The numerical method can properly predict the critical location in structure of early age cracking. If proper measures are taken, the crack may be avoided.
• The stress result of the analysis method surpasses ultimate tensile strength, but no macro crack occurs in practice. Because the concrete softening is neglected in this method. For accurately calculating stress in the early age, concrete damage should be considered.

REFERENCES