RELIABILITY ANALYSIS AND DESIGN OF GFRP-REINFORCED BRIDGE DECKS

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Abstract

Glass-fiber reinforced polymer (GFRP) reinforcement is being used in bridge decks as a replacement for steel reinforcement. It is thought that the GFRP reinforcement does not corrode and can be a more sustainable material for reinforced concrete structures. The design specifications allow the use of GFRP-reinforced concrete and require that the design tensile strength of the GFRP reinforcement be a function of an environmental factor and the guaranteed ultimate tensile strength (GUTS). The GUTS is defined as the mean tensile strength of the unexposed, newly produced reinforcement minus three standard deviations of the test lot. However, limited research has been performed to quantify the time-variant capacity of GFRP reinforcement embedded in concrete and thus the reliability of the environmental factor is unknown.

Using a Bayesian approach, a probabilistic model for the tensile capacity of GFRP reinforcement has recently been developed. The model uses tensile capacity data of GFRP reinforcement embedded in concrete for up to 7 years. This model is used herein to assess the time-variant structural reliability of a bridge deck. The results on the ability of GFRP-reinforced bridges to withstand future loads can be used to estimate the service life and optimize the design and construction for sustainable infrastructure systems.

1. INTRODUCTION

GFRP reinforcement has been identified as having non-corrosive characteristics, high tensile strength, and high strength to weight ratios. The use of GFRP reinforcement has increased significantly in many applications, including bridge decks, pavements, and walls. However, there is still a reluctance to use GFRP reinforcement, likely due to the lack of long-term performance data on GFRP reinforcement embedded in concrete.

Although GFRP reinforcement does not exhibit “classical” corrosion, many publications have reported that there is a significant reduction in the tensile capacity of GFRP reinforcement when exposed to moisture and/or alkaline solutions [1-3] or embedment in concrete [4-6].
Based on many short-term and accelerated exposure tests, degradation models have been developed to predict the longer-term performance of GFRP reinforcement. However, significant debate exists on the recommended models and the limits published in the design codes. For example, the rate and ultimate amount of this reduction is unknown, yet ACI Committee 440.1R (2007) and AASHTO LRFD Specifications (2008) require using an environmental reduction factor as a design parameter that considers the reduction in the tensile strength of GFRP in actual structures [7-8]. This reduction factor, \( C_E \), is 0.8 for concrete not exposed to earth and weather and 0.7 for concrete exposed to earth and weather. The design tensile strength, \( \sigma_{\text{ACI440}} \), of GFRP reinforcing bar considering these required reductions is defined as 
\[
\sigma_{\text{ACI440}} = C_E f_{\text{fu}}^*,
\]
where \( f_{\text{fu}}^* \) is the guaranteed ultimate tensile strength (GUTS) of the GFRP bars. The debate on the long-term performance of the GFRP reinforcing bars is a direct result of using models based on short-term and accelerated exposure data. However, a “valid” prediction model should include influencing parameters. If the GFRP has less capacity than expected in the current codes, the design process could be challenged. In addition, the influence of the reduced bar capacity on the structure capacity and reliability needs to be assessed.

Trejo et al. (2009, 2010) and Gardoni et al. (2010) developed a probabilistic model to estimate the residual tensile capacity of GFRP bars as a function of time. The model was developed using GFRP bars embedded in concrete for 7 years and shorter-term data for GFRP reinforcement embedded in concrete from the literature. The model indicates that GFRP reinforcement bars with larger diameters exhibit lower rates of capacity loss [9-11]. Using this probabilistic model, this paper assesses the time-variant structural reliability of a typical bridge deck reinforced with GFRP bars. GFRP reinforced decks containing two different sizes of the GFRP bars were designed and assessed to evaluate the time-dependant capacity and reliability of the bridge deck as a function of the bar size.

2. TIME VARIANT CAPACITY OF GFRP BARS EMBEDDED IN CONCRETE

Trejo et al. (2009) and Gardoni et al. (2010) developed the following probabilistic model to predict the capacity of GFRP bars embedded in concrete over time, \( t \) [9-10]:
\[
\sigma_t(x_b, \Theta_b) = \left(1 + s_0 \cdot e_0\right) - \lambda \left(\frac{D \cdot t}{R_0^2}\right)^\alpha \left(1 + s \cdot e\right) \cdot \mu_{\sigma_0}
\]
(1)
where \( x_b = (D, R_0) \) is a vector of basic variables, \( D \) is the diffusion coefficient, \( R_0 \) is the radius of the GFRP bar at \( t = 0 \), \( s_0 \cdot e_0 \) is an error term that captures the variability of the strength at \( t = 0 \) around its mean \( \mu_{\sigma_0} \), \( s \cdot e \) is an error term that captures the variability in the reduction term \( \lambda (D \cdot t / R_0^2)^\alpha \), \( e_0 \) and \( e \) are statistically independent normally distributed random variables with zero mean and unit variance (normality assumption), \( s_0 \) and \( s \) are the constant standard deviations of the two error terms (homoskedasticity assumption), and \( \Theta_b = (\lambda, \alpha, s_0, s) \) is a vector of unknown parameters introduced to fit the data.

A Bayesian approach was used to estimate the statistics (means, variances, and correlation coefficients) of the unknown parameters based on long-term exposure data (up to 7 years of exposure to actual environmental conditions) on 16M (#5), and 19M (#6) GFRP bars from three different manufacturers. Table 1 lists the posterior statistics of \( \Theta_b \) [9-10].
Table 1: Posterior statistics of the unknown parameter [9-10]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Correlation Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$</td>
<td>0.135</td>
<td>0.011</td>
<td>1 - 0.84 - 0.04 - 0.28</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.207</td>
<td>0.082</td>
<td>-0.84 1 0.04 - 0.25</td>
</tr>
<tr>
<td>$s_0$</td>
<td>0.039</td>
<td>0.003</td>
<td>-0.04 0.04 1 - 0.02</td>
</tr>
<tr>
<td>$s$</td>
<td>0.557</td>
<td>0.043</td>
<td>-0.28 - 0.02 -0.25 1</td>
</tr>
</tbody>
</table>

3. TIME-VARIANT CAPACITY AND RELIABILITY OF GFRP-REINFORCED BRIDGE DECKS

Because GFRP bars have high tensile strength and exhibit brittle failure modes, the design of members with GFRP reinforcement uses a different design philosophy than that of steel reinforced members [7, 12]. To prevent catastrophic failure of the deck, the deck is over-reinforced to ensure concrete crushing failure rather than bar yielding. The nominal flexural strength is estimated based on strain compatibility, load equilibrium, and possible failure modes (either concrete crushing or bar rupture failure mode) [12]. Bridge decks are typically designed and analyzed using a unit-wide strip deck model with rectangular cross-section.

In this paper, the constitutive model of concrete is modeled as the MacGregor’s parabolic stress-strain relationship with strain ranging from the maximum tensile and the ultimate strain (herein 0.0035) [13]. The modulus of elasticity of concrete, $E_c$, is modeled using the probabilistic model developed by Gardoni et al. (2007) [14]. The constitutive model for the GFRP bar is assumed to be a linear elastic stress-strain relationship with a constant modulus of elasticity, $E_f$. The ultimate stress and strain are determined by the time-variant capacity model in Eq. (1). To consider the degradation and failure of each individual bar, the load distribution concept is used. Each bar is considered to be a random variable and the capacity of each bar is summed to calculate the moment capacity of the section as follows:

$$f_c(\varepsilon) = \left[1.8f'_c(\varepsilon/e_{\varepsilon_c})\right] \left[1+(\varepsilon/e_{\varepsilon_c})^2\right]$$

(2)

where $f'_c$ is actual compressive strength [MPa (ksi)], $f_c(\varepsilon)$ is the compressive stress at the compressive strain, $\varepsilon$ [MPa (ksi)], $e_{\varepsilon_c} = 1.7 f'_c/E_c$ is the peak strain.

The moment capacities of a bridge deck can be estimated using the time-variant bar capacity model, the dominant failure mode, strain compatibility, and force equilibrium. The following equation can be used to determine the capacity of a deck at time $t$:

$$C_t(x, \Theta) = \min\left[C_{CF,t}, C_{BF,t}\right]$$

(3)

where $C_{CF,t}$ is the moment capacity of the deck when the concrete crushing failure occurs, $C_{BF,t}$ is the moment capacity of the deck when the GFRP bar failure occurs, $x = (x_c, x_d)$, $x_c = (f'_c, E_c, e_{\varepsilon_c}, b, h, d, A_f, R_0, \sigma_{t(i)}, E_f, \varepsilon_f, n)$, and $A_f$ is the area of the GFRP reinforcement in the given section [mm$^2$ (in.$^2$)], $\varepsilon_f$ is the strain of the GFRP reinforcement at the concrete crushing strain in the top fiber, $e_{\varepsilon_c}$, $b$ is the width of the cross section [mm (in.)], $d$ is the distance from the extreme compression fiber to the centroid of tension reinforcement [mm (in.)], $h$ is the depth of the section [mm (in.)], $\sigma_{t(i)}$ is the capacity at time $t$ (years) of the $i$th
individual GFRP bar, and \( n \) is the number of GFRP bars provided in the given section \( (n = b / s) \), where \( s \) is the bar spacing, and \( \Theta = (\Theta_b, \Theta_E) \), where \( \Theta_E \) is a vector of unknown parameters introduced to fit the MOE data [14].

When the concrete crushing failure is dominant \( (C_t = C_{CF,t}) \), the moment capacity can be estimated as follows:

\[
C_t(x, \Theta) = C_{CF,t} = A_f E_f \varepsilon_f (d - c + \bar{y}) + \sigma e
\]

where \( c \) is the distance measured from the top extreme fiber to the neutral axis, \( \bar{y} \) is the distance from the neutral axis to the location of the resultant compression force, and \( \varepsilon_f \) is the strain in the GFRP reinforcement when the concrete strain reaches crushing failure. The strain in the GFRP bar can be determined using the following equation:

\[
\varepsilon_f = 0.5 \left[ \varepsilon_{cu}^2 + \left( 4bd \right) \left( E_f A_f \right) \int_0^{\varepsilon_{cu}} f_c(\varepsilon) d\varepsilon \right]^{0.5} - 0.5 \varepsilon_{cu} \leq \varepsilon_{fu}
\]

When the bar failure is dominant \( (C_t = C_{BF,t}) \) the maximum strain in the concrete, \( \varepsilon_c \), is smaller than ultimate strain, \( \varepsilon_{cu} \). To determine the concrete strain, \( \varepsilon_c \), strain compatibility and force equilibrium conditions can be used:

\[
\begin{align*}
\left( \sum_{i=1}^{n} \sigma_{i(i)} / n \right) & - \left( \sum_{i=1}^{n} \sigma_{i(i)} / n E_f + \varepsilon_c \right) A_f - b d f_c e_c \left( 0.9 \ln \left[ 1 + \left( \varepsilon_c / \varepsilon_{cu} \right)^2 \right] \right) / \left( \varepsilon_c / \varepsilon_{cu} \right) = 0
\end{align*}
\]

Note that Eq. (6) is only valid when \( \varepsilon_c \leq \varepsilon_{cu} \) and \( c \leq c_b = \varepsilon_{cu} d / (\varepsilon_{cu} + \varepsilon_{fu}) \). In this case, \( C_t \) can be estimated using Eq. (4) with \( \varepsilon_f = \varepsilon_{fu} \).

### 3.1 Assessment of the Deck Fragility

Fragility is defined as the conditional probability of not meeting a specified performance level for a given moment demand, \( D \). In assessing the fragility, a limit state function \( g(\cdot) \) is introduced such that the event \( \{ g(\cdot) \leq 0 \} \) denotes not meeting the specified performance level. Using the probabilistic model for the time-variant capacity of a GFRP deck described in Eq.(3), a limit state function is written as:

\[
g_t(x, \Theta) = C_t(x, \Theta) - D
\]

The fragility at any time \( t \) is then written as:

\[
F_t(D, \Theta) = P\left[ g_t(x, \Theta) \leq 0 \mid D \right]
\]

The uncertainty in Eq. (8) arises from the inexact nature of the model \( C_t(x, \Theta) \) and its nested models captured by \( \varepsilon_0 \) and \( \varepsilon_\epsilon \), the inherent randomness (or aleatory uncertainty) in \( x \), and the statistical uncertainty in \( \Theta \). Figure 1 shows a conceptual three-dimensional plot of \( F_t(D, \Theta) \) as a function of \( D \) and \( t \). Following Gardoni et al. (2002), two estimates of \( F_t(D, \Theta) \) are possible based on different treatments of the parameter uncertainties: point and predictive estimates.

A point estimate of \( F_t(D, \Theta) \) ignores the epistemic uncertainties in the model parameters \( \Theta \) and uses a point estimate \( \hat{\Theta} \) (e.g., the posterior mean of \( \Theta \)) in place of \( \Theta \),
i.e., \( \hat{F}_t(D) = F(D, \Theta) \). A predictive estimate is obtained as the expected value of \( F_t(D, \Theta) \) over the posterior distribution of \( \Theta \), \( f_\Theta(\Theta) \), i.e.,

\[
\hat{F}(D) = \int_{\Theta} F(D, \Theta) f_\Theta(\Theta) d\Theta
\]

(9)

where the epistemic uncertainties are incorporated into the predictive estimates of the fragility in an average sense. Eq. (9) typically needs to be solved numerically.

**Figure 1:** Conceptual plot of as a function of \( D \) and \( t \)

### 4. DECK DESIGN APPLICATION

In this section, the probability models to predict the GFRP bar and deck capacities are used to estimate the fragility curve of a typical GFRP-reinforced bridge deck designed in accordance with the AASHTO specifications (2000) [15].

#### 4.1 Capacity for Example of GFRP-Reinforced Bridge Deck

A GFRP reinforced bridge deck from the Morristown Bridge is used to analyze the time-variant capacity [16]. The Morristown Bridge is located in Morristown, Vermont, USA. It has five 43 m (141 ft) long steel girders. The deck is a 230 mm (8 in.) thick concrete slab with 2.36 m (7.74 ft) girder spacing. The deck was designed according to the AASHTO specification (AASHTO 2000) and the American Concrete Institute (ACI) GFRP design guidelines (ACI 440.1R-01) [8]. Based on the serviceability criteria [maximum crack width = 0.5 mm (0.02 in.)], 16M (#5) GFRP reinforcement with 100 mm (4 in.) spacing was recommended for the bottom layer in the transverse direction. The overhang section required 19M (#6) GFRP reinforcement with 100 mm (4 in.) spacing for the top layer in the transverse direction. For constructability reasons, the final design of the bridge had the top and bottom GFRP reinforcement layers with 19M (#6) GFRP reinforcement. A detailed design procedure and drawings are provided in Benmokrane et al. (2006)[16]. In this study, only the deck section between the girders is analyzed. Table 2 presents two different design alternatives. Design I uses 19M (#6) GFRP reinforcement to achieve the same nominal moment in accordance with AASHTO and ACI design [16]. Design II uses 13M (#4) GFRP reinforcements to produce the same nominal moment as Design I.

Table 3 shows the input parameters for assessing the probability of failure for the GFRP reinforced deck. The standard deviation of the GFRP reinforcement and the concrete properties are estimated based on Nowak and Szerszen (2003) and Kulkarni (2006) [17-18]. The statistics
of the capacities at $t=0$, the elasticity values, and the diffusion coefficients of GFRP are obtained from Trejo et al. (2005, 2009) [9, 19].

Table 2: Design of GFRP reinforced deck

<table>
<thead>
<tr>
<th>Design Alternatives</th>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design I</td>
<td>19M (#6)</td>
<td>9.53</td>
<td>0.0715</td>
<td>Lognormal (COV=0.75%)</td>
</tr>
<tr>
<td></td>
<td>$R_0$ (mm)</td>
<td>140</td>
<td>N.A.</td>
<td>Deterministic</td>
</tr>
<tr>
<td>Design II</td>
<td>13M (#4)</td>
<td>6.35</td>
<td>0.0476</td>
<td>Lognormal (COV=0.75%)</td>
</tr>
<tr>
<td></td>
<td>$R_0$ (mm)</td>
<td>67</td>
<td>N.A.</td>
<td>Deterministic</td>
</tr>
</tbody>
</table>

The flexural demand is estimated using the equivalent strip method (Article 4.6.2 in AASHTO 2007) [20]. The service limit moment demand, $D_S$, is calculated with the summation of unfactored dead load moment, unfactored live load positive moment including multiple presence factors, the dynamic load allowance factor (0.33), and the unfactored future wearing dead load moment. The as-built girder spacing of 2.36 m (7.75 ft) and the future wearing surface of 12.7 mm (0.5 in.) thickness are used for estimating the demand of 27.9 kN-m/m (6.27 kip-ft/ft). When the design moment, $D_D$, is considered, the demand is estimated to be 47.3 kN-m/m (10.6 kip-ft/ft).

Table 3: Parameters of GFRP reinforced deck

<table>
<thead>
<tr>
<th>Materials</th>
<th>Parameter</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>GFRP</td>
<td>$E_f$ (MPa)</td>
<td>38,470</td>
<td>4355</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>$(	ext{MPa})$</td>
<td>569</td>
<td>53</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>$D$ (m$^2$/sec)</td>
<td>$8.903 \times 10^{-13}$</td>
<td>$3.522 \times 10^{-13}$</td>
<td>Lognormal</td>
</tr>
<tr>
<td>Deck</td>
<td>$f'_c$ (MPa)</td>
<td>30.5</td>
<td>2.50</td>
<td>Lognormal</td>
</tr>
<tr>
<td></td>
<td>$c^u$ (mm/mm)</td>
<td>0.0035</td>
<td>N.A.</td>
<td>Deterministic</td>
</tr>
<tr>
<td></td>
<td>$c$ (mm)</td>
<td>38.1</td>
<td>N.A.</td>
<td>Deterministic</td>
</tr>
<tr>
<td></td>
<td>$h$ (mm)</td>
<td>228.6</td>
<td>N.A.</td>
<td>Deterministic</td>
</tr>
<tr>
<td></td>
<td>$d$ (mm)</td>
<td>$d=h-cR_0$</td>
<td>N.A.</td>
<td>Lognormal</td>
</tr>
</tbody>
</table>

5. FRAGILITY ESTIMATES

Because GFRP bars exhibits low ductility, typical GFRP reinforced bridges are over-reinforced to prevent catastrophic bar failure, and so that concrete crushing controls the failure. However, as the degradation of GFRP bars progresses the probability of GFRP bar failure in the deck increases.

As shown in Figure 2, the fragility curves of the two design examples are assessed at time, $t = 0$ and 75 years.
The fragility curves for Designs I and II are the same at $t = 0$. At time $t = 0$, the probability of failure of the deck is lower than $10^{-6}$ for $D = D_S$ and $D = D_D$. After 75 years the probability of failure increases significantly, in particular when 13M (#4) bars are used. For example, at the service moment demand, $D_S$, the probability of failure of a deck with 13M (#4) bars (0.025) is approximately 45% higher than that of a deck with 19M (#6) bars (0.005). For $D_D$, the probability of failure of a deck with 13M (#4) bars (0.044) is four times higher at 75 years when compared to the probability of a deck with 19M (#6) bars (0.011). This research indicates that bridge decks reinforced with 13M (#4) GFRP bars and 19M (#6) GFRP bars have probabilities of failure of 0.025 and 0.005. The generally accepted probability of failure for bridge structures based on the AASHTO LRFD is 0.001. Both GFRP-reinforced decks evaluated here exceed the generally accepted AASHTO probability of failure limit after 75 years of exposure to environmental conditions. This indicates that the environmental exposure factor, $C_E$, likely does not adequately account for the expected degradation of the GFRP bars at later ages. Therefore, consideration should be given to reducing the environmental exposure factor for exterior exposure conditions.

6. CONCLUSIONS

GFRP reinforcing bars have been identified as potential alternatives to steel reinforcement. ACI and AASHTO provide procedures for designing reinforced concrete structures with this type of reinforcement. Although significant research has been performed on the time-variant tensile capacity of GFRP reinforcing bars exposed to solutions, until recently only short-term studies have been performed to assess the time-variant tensile capacity of GFRP reinforcing bars embedded in concrete. This paper uses a newly developed model for the tensile capacity of GFRP reinforcement developed using data on GFRP embedded in concrete for up to 7 years [9-11] to assess the time-variant capacity of two separate bridge decks reinforced with different sizes of GFRP reinforcing bars. Results from the analysis indicate that larger diameter GFRP reinforcing bars can provide lower probabilities of failure when embedded in concrete. As the GFRP diameter decreases, the probability of failure of the deck increases. The analysis also indicates that the probability of failure of the decks containing both 13M (#4) and 19M (#6) GFRP bars exhibit higher probability of failures than failure limits generally accepted by AASHTO. Reducing the environmental exposure factor, $C_E$, for GFRP-reinforced bridge decks (i.e., exterior exposure conditions) may be warranted.
REFERENCES


