MECHANICAL PROPERTIES OF CONCRETE WITH SAP
PART II: MODULUS OF ELASTICITY

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Abstract
In this study, focus is on the modulus of elasticity for concrete with superabsorbent polymers (SAP). The results show that based on composite theory it is possible to establish a model, which predicts overall concrete elasticity. The model assumes a three phase material of aggregate, cement paste, and air with volume fractions of the three phases as well as elastic properties of paste and aggregates as input parameters. Addition of SAP changes the E-modulus, because it both has an influence on properties of the cement paste and on the volume of air voids. Here, the E-modulus is an example of a mechanical property, and the same methodology can probably be applied to other mechanical properties. It is often assumed that a range of mechanical properties of concrete can be derived if the compressive strength is known. The link between the compressive strength and other mechanical properties is often a more or less empirical relation. The results show that when introducing SAP, models of a more empirical nature can be misleading (and e.g. relations stated in codes are often of this empirical nature). The reason is twofold: First, the empirical models often have a general problem with the effect of air voids. Second, SAP addition may at the same time lead to increased compressive strength (as shown in [5]) and reduced E-modulus. A prediction based solely on compressive strength therefore overrates the modulus of elasticity, so the empirical models are unsafe to use for concrete with SAP

1. Introduction

Super absorbent polymer (SAP) has been introduced in concrete mix design as a means of internal water curing of cementitious materials with low w/c ratios [1]. However, before SAP can be used in concrete production on a larger scale, it is important to clarify how SAP influences other concrete properties. The aim of this paper is to make a systematic study on how addition of SAP influences modulus of elasticity (also called E-modulus). Modulus of
elasticity for concrete with SAP has, to the knowledge of the authors, only been reported once before [2], and only for a limited number of concrete mixes.

2. Theory

2.1 Composite models

The modulus of elasticity of a two-phase material consisting of matrix and embedded particles can in principle be modelled in two ways [3], see table 1:

<table>
<thead>
<tr>
<th>Parallel model (Voight’s model)</th>
<th>Serial model (Reuss’ model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_m$</td>
<td>$\sigma_m = \sigma_p$</td>
</tr>
<tr>
<td>$\sigma_p$</td>
<td></td>
</tr>
</tbody>
</table>

Assumption: applied load results in equal strain in both phases

$$E = V_m \cdot E_m + V_p \cdot E_p$$  \hspace{1cm} (1)

$$\frac{1}{E} = \frac{V_m}{E_m} + \frac{V_p}{E_p}$$  \hspace{1cm} (2)

The parallel model predicts an upper limit, whereas the serial model predicts a lower limit for the E-modulus of the composite material. According to [3], the parallel model is the best approximate description, if the matrix has a higher E-modulus than the particles ($E_m > E_p$), whereas the serial model applies, when the opposite is true ($E_m < E_p$). How well the models fit reality depends among other things on the bond between matrix and particles [3].
In line with these principles, the elastic properties of a simple three-phase composite of cement paste aggregates, and air can be modelled as shown in figure 1. The hard aggregate particles are surrounded by a softer paste+air matrix, so these two phases are modelled in a series. In the paste+air phase, the soft air voids are surrounded by cement paste, so this is modelled in parallel.

\[
E_{\text{concrete}} = \left( \frac{V_{\text{agg}}}{E_{\text{agg}}} + \frac{V_{\text{paste+air}}}{E_{\text{paste+air}}} \right)^{-1} = \left( \frac{V_{\text{agg}}}{E_{\text{agg}}} + \frac{V_{\text{paste}} + V_{\text{air}}}{E_{\text{paste}} \cdot \frac{V_{\text{paste}}}{V_{\text{paste}} + V_{\text{air}}}} \right)^{-1}
\]

(3)

Figure 1: Suggestion for a simple model, which describes the elastic properties of concrete with air.

When modelling the development of E-modulus, the moduli of aggregate and air, \(E_{\text{agg}}\) and \(E_{\text{air}}\), are constants (\(E_{\text{air}} = 0\)), only \(E_{\text{paste}}\) changes due to hydration. Powers has suggested this to be modelled in the following way, see equation (4) [4, according to 3]:

\[
E_{\text{paste}} = A \cdot X^3
\]

(4)

where \(A\) is a constant and \(X\) is the gel space ratio. A similar approach was successfully used to model compressive strength in [5].

2.2 Empirical relations between compressive strength and E-modulus

There are several empirical formulas, which relate the compressive strength of concrete to the modulus of elasticity, e.g.

- DS 411 [6]:
  \[
  E = 51 \text{ GPa} \frac{f_{ck}}{f_{ck} + 13 \text{ MPa}}
  \]
  (5)

- CEB-FIP [7]:
  \[
  E = 21.5 \text{ GPa} \left( \frac{f_{ck}}{10 \text{ MPa}} \right)^{1/3}
  \]
  (6)
3. Materials and methods

The test programme comprises 3 test series with different w/c ratios (0.35, 0.40, and 0.50, respectively). Each test series comprises 5 different levels of SAP addition from 0.0 to 0.6% of cement weight. The mix without SAP is a reference mix. The rest of the mixes in the test series have the same cement content as the reference mix. The water content is the sum of water in the reference mix and water assumed to be absorbed by SAP during mixing (12.5 g water pr. g dry SAP). The amount of aggregates is adjusted to maintain constant volume. For more details on materials, mix design, making of test specimens, curing conditions etc., see [5].

Modulus of elasticity is determined from the stress vs. strain relation registered during measurements of compressive strength. Results for compressive strength have been reported in [5]. Measurement of compressive strength follows EN 12390-3 [8], whereas measurement of modulus of elasticity does not follow a standardised method. Measurements of deformation are made with transducers, which are removed before the cylinder breaks. The transducer registers deformation over 100 mm at the centre part of a Ø100 x 200 mm cylinder, see figure 2.

Figure 2: Left: Test specimen with deformation transducers. Right: Example of stress vs. strain curves after different hardening times (here concrete with w/c = 0.50 and no SAP added).

Concrete is not a linear elastic material, so the modulus of elasticity depends on the actual load level. In e.g. DS 423.25 [9], E-modulus is defined as the slope of the secant between $\sigma_{\text{min}} = 0.5$ MPa and $\sigma_{\text{max}} = 0.4 \cdot f_\text{c}$, where $f_\text{c}$ is the compressive strength. However, during processing of data from this study, a problem was discovered with consistency between stress and strain measurements at low load levels. Therefore, the following definition of E-modulus
is used, as this gives robust and repeatable measurements: E-modulus is defined as the slope of the secant between $\sigma_{\text{min}} = 0.1 \cdot f_c$ and $\sigma_{\text{max}} = 0.4 \cdot f_c$.

Measurements of E-modulus are carried out 1, 2, 3, 7, and 28 days after casting to follow the development of E-modulus. Each time measurements are carried out for 3 cylinders. As a supplement, measurements of degree of hydration are made with paste samples, see [5].

4. Results

Figure 3 shows the measured E-moduli:

![Figure 3: E-modulus vs. gel space ratio (calculated according to Powers’ model from w/c ratio and degree of hydration [5]).](image)

The gel space ratio is a measure of the denseness of the paste phase excluding air voids. It is a function of w/c ratio and degree of hydration; it increases, when the w/c ratio decreases and the degree of hydration increases. The results show that the E-modulus is higher, the higher the gel space ratio. This is to be expected owing to the definition of gel space ratio.

Results also show that the E-modulus is higher, the lower the air content. This is also expected.

5. Discussion

5.1 Test of model

Figure 3 is already an indication on how close the proposed composite model in figure 1 is to reality. In the model it is assumed that concrete mixes with equal gel space ratio have the same paste properties (paste phase not including air voids), though e.g. w/c ratios differ. At low gel space ratio, the paste phase is very elastic, and at high gel space ratio the paste phase
becomes stiffer, though it is still soft compared to the aggregates. When the paste phase becomes stiffer, i.e. obtains a higher E-modulus, the overall E-modulus for the concrete increases.

The model can also explain why the results spread out in a fan like formation. If the model had been a 3 phase parallel model, then 1% of air would simply reduce the concrete E-modulus 1%, no matter the elastic properties of the paste and aggregates, and like this the influence of air voids would be the same, no matter of the gel space ratio. But figure 3 shows that at low gel space ratio, the influence of air is low, whereas when the gel space ratio is high, the influence of air is more pronounced. This is to be expected from the model. If the paste phase is soft, it does not matter so much, if the paste+air layer is softened by the presence of air. But when the stiffness of the paste phase approaches the stiffness of the aggregate phase, then the softening effect of air is stronger.

The modulus of elasticity for the aggregates $E_{agg}$ is not known. The coarse aggregates are granite. E-modulus for granite is in the range 20-60 GPa [10]. It is assumed that $E_{agg}$ takes the value of 50 GPa, as it is not possible to make a robust regression for $E_{agg}$ and $A$ in equation (4) at the same time. A more accurate prediction requires direct measurement of E-modulus on specimens cut from the actual granite source (blue Rønne granite). Then a non-linear least square regression is made to determine $A$ and the exponent in formula (4).

Figure 4 shows a comparison of E-moduli predicted by the resulting model and measured E-moduli:

![Figure 4: Comparison of predicted and measured moduli of elasticity. See figure 3 for explanation of symbols.](image)

There seems to be a close fit between model predictions and measured values, except for two measurements, which may deviate due to experimental error. These two measurements were
omitted in the final regression. The exponent in formula (4) was determined to the value 1.89. This is significantly different from the value 3 proposed by Powers.

5.2 Importance of w/c and degree of hydration
The results show that properties of cement paste influence the E-modulus less than the compressive strength, see table 2:

Table 2: Compressive strength and modulus of elasticity measured for reference mixes with varying w/c ratios 28 days after mixing (figures in parenthesis are relative to relevant values for w/c=0.35, 0 % SAP).

<table>
<thead>
<tr>
<th>w/c</th>
<th>Compressive strength [MPa]</th>
<th>Modulus of elasticity [GPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35, 0% SAP</td>
<td>72.0</td>
<td>36.9</td>
</tr>
<tr>
<td>0.40, 0% SAP</td>
<td>66.8 (93%)</td>
<td>35.9 (97%)</td>
</tr>
<tr>
<td>0.50, 0% SAP</td>
<td>50.4 (70%)</td>
<td>33.1 (90%)</td>
</tr>
</tbody>
</table>

From composite theory, it is also expected that paste properties have less influence on the overall E-modulus of concrete than they have on compressive strength: Regarding compressive strength, the paste phase is the weak link, which is decisive for the concrete strength. Regarding E-modulus, the influence of paste properties depends on the volume fraction of the cement paste, which in this study is roughly \( \frac{1}{3} \) of the concrete volume. Of the concrete volume \( \frac{2}{3} \) is aggregates with elastic properties that are unchanged through the study.

As paste properties only to a less extent is reflected in the E-modulus, the contribution to E-modulus from SAP changing paste properties by increasing degree of hydration can almost be neglected.

5.3 Importance of void content
In figure 5 actual measurements of E-moduli for concrete mixes with w/c = 0.40 28 days after casting are compared with predictions made with DS 411 model (equation (5)) and CEB-FIP model (equation (6)). Both models predict E-moduli that are fare to high, probably because they are designed to take a characteristic compressive strength (i.e. lower 5 % or 10 % tail) as input parameter, where here an average of measurements is used as an input. The slopes of curves are different. The code models have low slopes indicating only a minor influence of air content on modulus of elasticity, whereas the measurements show a more pronounced reduction of E-modulus when the air content increases.

Figure 6 concerns a fictive type of concrete, which has compressive strength 40 MPa when the air content is 0 %. Boarder lines are shown for the DS 411 model and the CEB-FIP model, assuming a strength loss of 2.7% and 4.0% per % of air, respectively. The 2.7% corresponds to what was found in [5], whereas a value 4-5% is often assumed for less paste rich mixes [11]. Results are normalized, so they take the value 1, when there is no air in the concrete. The figure also shows the results for the composite model in figure 1, when assuming the
same E-modulus as predicted for the DS 411 model, when the air content is zero, and with 35 % paste and 65 % aggregates.

![Figure 5: Measured E-moduli for concrete mixes with w/c = 0.40 28 days after casting compared to values predicted by empirical DS 411 and CEB-FIP models (see equation (5) and (6), respectively).](image)

From the curve in figure 6, it can be observed that the presence of voids, i.e. SAP voids and naturally entrained air, reduces E-modulus more than what is accounted for in the empirical models based on compressive strength as sole input parameter. As the addition of SAP always creates voids, it is unsafe to use these models. Especially, as shown in [5], addition of SAP under certain circumstances increases the compressive strength, in which case the empirical relations predict an increased modulus of elasticity. Contrary to this, all the tested concrete mixes in this study showed that SAP addition reduced the modulus of elasticity, also the mixes with increased compressive strength.

It is not known why these models have a general problem with air voids. Maybe it is because much of the experimental work on linking compressive strength to other mechanical properties is made as early as in the 1920s and 1930s. At this time concrete was not air entrained, so the air content was limited and did not vary very much. By the time air entrainment became common practice (in Denmark in the 1970ies), the topic of relation between compressive strength and E-modulus was considered fully exhausted from a research point of view. But it is surprising that so little has been done to model air voids as a separate phase, considering that quite a lot has been done to model the interfacial transition zone as a separate phase in a 3 phase model, see e.g. [12].

As shown above, the contribution of SAP to void volume can explain its influence on E-modulus. However, it is important to point out that concerning this matter, SAP voids do not behave differently from other air voids. Therefore, when studying the influence of SAP on mechanical properties, it is important to register the total air content, not only content of SAP.
voids. Otherwise, it is very easy to draw wrong conclusions like “SAP reduces property X”, where SAP in fact may have no direct influence on property X. But if SAP e.g. changes workability in a way which leads to an increased air volume, it may indirectly change property X.

Figure 6: Comparison of model presented in figure 1 with DS 411 and CEB-FIP models for fictive type of concrete, see text for further explanation.

6. Conclusion

Regarding modulus of elasticity, it is possible to successfully model the influence of SAP, by assuming a 3 phase composite material with the phases: paste, aggregate, and voids.

This paper demonstrates that while theoretical models for mechanical properties work just as well for concrete with SAP as for concrete without, one has to be very careful with more empirical models. For example, it is a common rule of thumb that the modulus of elasticity follows the compressive strength - i.e. increased strength entails increased modulus of elasticity. However, SAP addition can at the same time result in higher compressive strength and lower modulus of elasticity, and for this reason it may be unsafe to use traditional formulas that relate compressive strength and modulus of elasticity.

References


