PROBABILISTIC STRENGTH PREDICTION OF ADHESIVELY BONDED JOINTS COMPOSED OF HETEROGENEOUS MATERIALS

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ABSTRACT: The strength prediction of adhesively bonded joints composed of heterogeneous fibre-reinforced materials such as composites and timber is difficult due to the anisotropic and brittle nature of the adherends, the complex stress distribution as well as the uncertainties regarding the associated material resistances. This paper describes a numerical model and a probabilistic method for the strength prediction of balanced double lap joints composed of the above mentioned adherends and epoxy adhesives. The method considers the scale sensitivity of material strength modelled using a Weibull statistical function, presents an explanation for the increased resistance of local zones subjected to stress peaks and allows safely predicting the strength of adhesively bonded joints composed of brittle adherends. Finally, the paper shows how probabilistic methods have to be formulated in the framework of current codes.

1 INTRODUCTION

1.1 Adhesively bonded joints

Adhesively bonded joints are increasingly being used for orthotropic fibrous brittle materials such as fibre-reinforced polymers (FRP) [Kel05; Val06a; Val06b] and timber [Heh10]. Attempts to predict the strength of such joints on a stress basis usually failed unless particular adaptation factors are introduced, since the load transfer in is characterized by huge stress peaks at the end of the splices [Val06a].

For adhesive bonding to gain larger acceptance for the use in load-carrying structures, methods have to be developed that predict joint strength as a function of the geometry, the material properties, and the type of loading. Besides these mechanical considerations, it is also necessary to formulate a safety concept that is coherent with the established partial safety factor concept. These methods usually involve the definition of characteristic design values, for the loads and the resistances, which are most commonly based on quantile-values of the statistical distributions: the characteristic value $X_k$ of an (usually experimentally gathered) dataset $\{X_i\}$ is defined as being the value for which the probability that the random variable $X$ will be bigger than a given value $X_k$ is 5%.

It is often left unsaid that most techniques used to define characteristic values assume that the data is normally distributed; however, when it comes to brittle materials, it is largely known that the statistics are best described using alternative distributions, as for example the Weibull distribution. Consequently, actual procedures to derive characteristic values should be studied in this regard; the paper presented herein suggests such a procedure for adhesively bonded joints composed of heterogeneous materials.
1.2 Probabilistic strength prediction methods

To overcome the limitations of the stress based strength prediction, a probabilistic strength prediction method was investigated for adhesively bonded joints composed of FRP [Val06b] and timber [Heh10]. The prediction method considers the scale sensitivity of the material strength, considering not only the magnitude of the stress fields, but also the volume, over which they act. Weibull-Theory was used to model the material strength and a general good agreement was obtained between strength predictions and failure loads of joints composed of brittle adherends and epoxy adhesive layers. For a general overview on size effects and its relations to strength, the reader is kindly redirected to [Baž05]. For the purpose of this publication, the following is reminded: (1) probabilistic strength prediction methods assume that the investigated material exhibits brittle failure; (2) the material strength is then usually statistically described as being Weibull-distributed.

Further, for the implementation of any strength prediction method, including the ones based on probabilistic concepts, a failure criterion for the material in needed. Failure criteria ordinarily used for isotropic materials do not apply to orthotropic and anisotropic materials and their use usually result in incorrect stress state interpretation. There is extensive literature investigating to which extend failure criteria derived from theoretical consideration are applicable to FRP [Kel05] and timber [Kas05].

2 EXPERIMENTAL INVESTIGATION

2.1 Specimen description

Symmetrical double-lap joints with rectangular sections were tested. The joints consisted of two outer and two inner adherends, being either timber or FRP, connected by a layer of adhesive, a commercial two-component epoxy adhesive, SikaDur330. Table 1 lists the mechanical properties of all materials. The inner profiles were always twice as thick as the outer ones to keep the cumulative cross-section constant (Fig. 2.1).

In the frame of the investigations presented herein, four experimental series are presented:

- Series 1: FRP joints with varied overlap length, \( L \), 50 mm, 100 mm, 150 mm, 200 mm. The other parameters were kept constant with: \( t_o=5 \text{ mm}, t_i=10 \text{ mm}, t_a=10 \text{ mm} \).
- Series 2: FRP joints with varied adhesive thickness, \( t_a \), 1 mm, 2 mm, 3 mm, 5 mm, 10 mm. The other parameters were kept constant with: \( t_o=5 \text{ mm}, t_i=10 \text{ mm}, L=100 \text{ mm} \).
- Series 3: Timber joints with varied overlap length, \( L \), 40 mm – 280 mm in steps of 40 mm. The other parameters were kept constant with: \( t_o=19 \text{ mm}, t_i=38 \text{ mm}, t_a=1.5 \text{ mm} \).
- Series 4: Timber joints with varied adhesive thickness, \( t_a \), 0.5 mm, 1.0 mm, and 1.5 mm. The other parameters were kept constant with: \( t_o=19 \text{ mm}, t_i=38 \text{ mm}, L=160 \text{ mm} \).

![Fig. 2.1](image-url) Geometry of lap joint specimens (not to scale); used to introduce the nomenclature.
2.2 Mechanical properties of FRP

Specimens were manufactured from pultruded GFRP flat profiles, 350×50×5 mm and 350×50×10 mm, and consist of E-glass fibres embedded in an isophthalic polyester resin. The fibre architecture comprised essentially unidirectional rovings towards the centre and, depending on the profile thickness, one or two combined mats towards the outside. The combined mats consisted of chopped strand mats (CSM) and woven fabrics 0°/90° of different weights, both stitched together. The outer surfaces were covered by a protective polyester surface veil (40 g/m²). The mechanical properties of the profiles, listed in Table 1, were determined through full-scale tensile tests. The profiles showed an almost linear behaviour up to failure. The strength of the material used herein was determined using a shear-tensile interaction device [Val06b], which allows measuring material strength values under any combination of through-thickness tensile and shear stresses. For the representative inner adherends material, which triggered rupture of the joints, the failure criterion represented by Eq. (2.1) was found to be sufficiently accurate.

\[
\frac{\sigma_Y^2 + \tau_{XY}^2}{f_1^2 f_{12}^2} = 1
\]

where
- \(\sigma_Y\) through-thickness stress
- \(\tau_{XY}\) shear stress
- \(f_1\) average value through-thickness tensile strength
- \(f_{12}\) average value shear strength

2.3 Mechanical properties of timber

The timber used was Spruce (Picea Abies) cut from high quality defect-free boards. The elastic properties of the timber required for the numerical investigations (longitudinal modulus of elasticity \(E_1\), the transverse modulus of elasticity \(E_2\) and the shear modulus \(G_{12}\) were determined on small clear specimens. Table 2.1 summarizes the average values; these values are higher than those stated in the literature [Gre03], which is explained by the use of high quality timber. The material strength of the timber was characterized based upon the Norris failure criterion [Nor50] given by Eq. (2.2):

\[
\frac{\sigma_X^2 - \sigma_X \sigma_Y + \sigma_Z^2 + \tau_{XY}^2}{f_1^2 f_2^2 f_{12}^2} = 1 \quad \frac{\sigma_X^2}{f_1^2} = 1 \quad \frac{\sigma_Z^2}{f_2^2} = 1
\]

where
- \(\sigma_X, \sigma_Y, \tau_{XY}\) stresses
- \(f_1, f_2, f_{12}\) strength parameters relatively to the grain axes (1,2)

To determine the strength parameters, tests were performed on dog-bone shaped specimens exhibiting different orientations, \(\alpha\), relatively to the grain. For a description of the method, please refer to [Heh10; Xav04], where the procedure followed is explained in detail.
Table 2.1. Mechanical material properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>$E_X$ [MPa]</th>
<th>$E_Y = E_Z$ [MPa]</th>
<th>$f_1$ [MPa]</th>
<th>$f_2$ [MPa]</th>
<th>$f_{1,2}$ [MPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>FRP</td>
<td>34,500</td>
<td>3,550</td>
<td>342</td>
<td>9.4</td>
<td>22.6</td>
</tr>
<tr>
<td>Timber</td>
<td>17,910</td>
<td>1,120</td>
<td>98.2</td>
<td>4.5</td>
<td>16.5</td>
</tr>
<tr>
<td>Adhesive</td>
<td>4,563</td>
<td></td>
<td>39</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2.4 Experimental results

All investigated specimens of adhesively bonded joints, featuring FRP or timber adherends, failed in a brittle manner by splitting just below the end of the overlap; in all cases failure was triggered by a crack that developed from the surface. Fig. 2.2 illustrates typical failure.

![Double-lap specimens after testing: FRP (left) and timber (right).](image)

Two different values were used to characterize the experimental results for the test series:

1. The statistical mean value of the individual test results for each parameter value varied, and labelled herein $F_{E,m}$.

2. The strength corresponding to the 5%-quantile value of the corresponding data set $F_{E,k}$, assuming a normal distribution, which conceptually corresponds to the definition of a characteristic value according to the majority of codes.

The formal definition of the characteristic value according to codes involves additional factors, but their introduction does not affect the outcome of the conceptual conclusions drawn herein. The above defined strengths are graphically reported in Figs. 3.2 and 3.3.

3 PROBABILISTIC STRENGTH PREDICTION

3.1 Numerical investigation

All joints were modelled using the finite element program ANSYS (v11). Both the FRP and the timber were modelled as linear-elastic orthotropic materials; the adhesive was modelled as being a linear-elastic isotropic material. The reader is kindly redirected to [Val06b] for the stress profiles of the joints involving FRP and [Heh10] regarding timber. Adhesively bonded joints typically exhibit sharp $\sigma_Y$- and $\tau_{XY}$-stress peaks at both ends of the overlap length, represented by Fig. 3.1 for the timber joints. The reader is kindly redirected to [Val06b] for the stress profiles of the joints involving FRP.
3.2 Concept of the strength prediction

As stress-based approaches, due to the huge stress peaks at the ends of the overlaps, cannot predict the strength of adhesively bonded joints, a probabilistic method was pursued herein. The principles of this method were presented in detail in [Val06b], so that these are only summarized herein. Idealizing the joints under consideration as being constituted by \( n \) elements that could potentially fail, its survival depends on the simultaneous non-failure of all elements \( i \leq n \). As a result, for a given applied load, \( F \), the probability of survival of the joint can be calculated by Eq. (3.1):

\[
P_S(F) = \prod_{i=1}^{n} P_{S,i}(F)
\]

where \( P_S \) probability of survival of joint
\( P_{S,i}(F) \) probability of survival of element \( i \) associated with the applied load \( F \)

Most commonly, \( P_S \) is expressed by a Weibull distribution, which assumes that the material under consideration exhibits a brittle failure. Herein a two-parameter Weibull distribution, as defined by Eq. (3.2), is considered:

\[
P_S = \exp\left(-\int \frac{\sigma}{m}^k dV\right)
\]

where \( \sigma \) stress acting over a volume \( V \)
\( m \) is the characteristic stress or scale parameter
\( k \) shape parameter that gives a measure of the strength variability

Conversely, for two volumes \( V_1 \) and \( V_2 \) exposed to constant stresses \( \sigma_1 \) and \( \sigma_2 \) at failure, statistical size-effects can be simply expressed using Eq. (3.3):

\[
\frac{\sigma_1}{\sigma_2} = \left(\frac{V_2}{V_1}\right)^{\frac{1}{k}}
\]
The subsequent determination of the Weibull-parameters for FRP and timber yielded in the following values: \( k = 17.37 \) and \( m = 1.028 \) for FRP, and \( k = 3.72 \) and \( m = 1.124 \) for timber. Herein the failure criteria of FRP, Eq. (2.1), and timber, Eq. (2.2), can be seen as stress operators that control the failure of the respective materials. For the following consideration, a failure function \( \phi_{F,i} \) is defined by reformulation of Eqs. (2.1 and 2.2), leading to Eqs. (3.4) for FRP and (3.5) for timber:

\[
\phi_F^2 = \frac{\sigma_X^2}{f_1^2} + \frac{\tau^2_{X,Z}}{f_{1,2}^2} 
\]

(3.4)

\[
\phi_F^2 = \frac{\sigma_X^2}{f_1^2} - \frac{\sigma_X \sigma_Z}{f_1 f_2} + \frac{\tau^2_{X,Z}}{f_{1,2}^2} 
\]

(3.5)

Consequently, if each constituent element \( i \), with a volume \( V_i \) is subjected to a constant value of the failure function \( \phi_{F,i} \), \( P_S \) of the whole member is given by Eq. (3.6):

\[
P_S = \prod_{i=1}^{n} \exp \left[ - \frac{V_i}{V_0} \left( \frac{\phi_{F,i}}{m} \right)^k \right] = \exp \sum_{i=1}^{n} \left[ - \frac{V_i}{V_0} \left( \frac{\phi_{F,i}}{m} \right)^k \right]
\]

(3.6)

### 3.3 Application

Eq. (3.6) can be implemented in a post-processing routine for numerical results and the strength of adhesively bonded joints can then be predicted. Two different strengths were numerically determined using the probabilistic method and are displayed in Figs. 3.2 and 3.3:

(3) Failure loads were defined herein as being the point of equal probability of survival or failure, i.e. \( P_S = 0.5 \), which in a first approach designates the value \( F \) for which half of the specimens would survive. The corresponding value is the predicted strength \( F_{P,m} \).

(4) To determine a strength value used by practitioners for design purposes, the strength value corresponding to a probability of failure of 5\% and labelled herein \( F_{P,k} \), here the Weibull distribution was applied.

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**Fig. 3.2.** Experimental and predicted joint strength vs. \( L \) (left) and \( t_a \) (right), for FRP.
Fig. 3.3. Experimental and predicted joint strength vs. $L$ (left) and $t_a$ (right), for timber.

4 CONCLUSIONS

The experimental and numerical results of this study clearly indicate the following facts:

- Joint strengths increase with the overlap length, with a tendency to a flattening towards the long overlaps (as shown on the timber and FRP).
- Joint strength also decreases with adhesive layer thicknesses (shown on the FRP).
- Joint strengths can be predicted using a probabilistic method; the accuracy can be qualified as very good for both, the FRP and the timber joints.

Furthermore, regarding the implementation of the described probabilistic method for modern codes and standards which are based on safety factors derived from statistical considerations, it can be stated that:

- When dealing with highly brittle components, the failures, rather than being normally distributed, tend to be best described by extreme value probability density functions, as for example the Weibull distribution.
- Considering that the joints fail in a brittle manner, Weibull statistics leads to a good agreement between experimentally and numerically determined characteristic strengths.
- It is thus recommended to consider Weibull distributions for the determination of characteristic strength values to be implemented in codes, if brittle failure is involved.
REFERENCES


